



Information Retrieval

Language Models

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Content of this Lecture

- Language Models
- Markov Models
- Data sparsity
- Language Models for IR

- Most material from [MS99], Chapter 6

Problem

- Given a prefix of a sentence: **Predict the next word**
 - “At 5 o’clock, we usually drink ...”
 - “tea” – quite likely
 - “beer” – quite unlikely
 - “a beer” – slightly more likely, but still
 - “biscuits” – semantically wrong
 - “the windows need cleaning” – syntactically wrong
- Similar to **Shannon’s Game**: Given a series of characters, predict the next one (used in communication theory)
- Abstract formulation: Given a language L and the prefix $S[1..n]$ of a sequence S , $S \in L$: **Predict $S[n+1]$**
- This is a **ranking problem** – no single solution

Applications

- Speech/character recognition
 - Given a transcribed prefix of a sentence – which word do we expect next?
- Automatic translation
 - Given a translated prefix of a sentence – what do we expect next?
- T9: "... information about common word combinations can also be learned ..."
- General: Use probabilities of next word as a-priori probability for **interpreting the next signal**
 - Helps to **disambiguate** between different options
 - Helps to make useful suggestions
 - Helps to point to likely errors (**observation \neq expectation**)

Language Models

- Problem: Learn a **model of a language** allowing predictions
- Classical approach: **Grammars**
 - Regular, context-free, ...
 - Grammars can be learned from examples
 - Not trivial, underdetermined, not covered here
 - Usually, **multiple continuations** of a prefix are allowed
 - (Deterministic) Grammars do not help in deciding which is the **most probable one**
 - Better: Probabilistic grammars
 - Probabilistic automata: Transitions have a relative frequency
- Grammars based on grammatical categories typically not fine grained enough: **Many equally probable** continuations remain

N-Grams over Words

- Popular and simple approach: **N-gram models**
 - “Indeed, it is difficult to beat a trigram model on the purely linear task of predicting the next word” [MS99]
- Definition

*A (word) **n-gram** is a sequence of n words.*
- Usage
 - Count frequencies of **all n-grams** in a corpus of the language
 - **Slide window** of size n over text and keep counter for each n-gram ever seen
 - Given a sentence prefix, predict **most probable continuation(s)** based on n-gram frequencies – how?

N-Grams for Language Modeling

- Assume a sentence prefix with $n-1$ words $\langle w_1, \dots, w_{n-1} \rangle$
- Look-up counts of all n -grams **starting** with $\langle w_1, \dots, w_{n-1} \rangle$
 - I.e., n -grams $\langle w_1, \dots, w_{n-1}, w_n \rangle$
- Choose that w_n whose n -gram is the **most frequent one**
- More formally
 - Compute, for every possibly w_n ,

$$p(w_n) = p(w_n \mid w_1, \dots, w_{n-1}) = \frac{p(w_1, \dots, w_n)}{p(w_1, \dots, w_{n-1})}$$

- Choose w_n which **maximizes** $p(w_n)$

Which n?

- In language modeling, one usually chooses $n=3-4$
- That seems small, but most **language effects are local**
 - But not all: “Dan swallowed the large, shiny, red ...” (Car? Pill? Strawberry?)
- Also, we cannot obtain **robust relative counts** for larger n - not enough training data
 - **Data sparsity** problem
 - In high dimensional problems, training data is always sparse

Content of this Lecture

- Language Models
- [Markov Models](#)
- Data sparsity
- Language Models for IR

History and Applications

- **Andrej Andrejewitsch Markov** (1856-1922)
 - Russian Mathematician
 - Developed Markov Models (or Markov Chains) as a method for analyzing language
 - Markov, A. A. (1913). "Beispiel statistischer Untersuchungen des Textes ‚Eugen Onegin‘, das den Zusammenhang von Ereignissen in einer Kette veranschaulicht (Original in Russisch)." *Bulletin de l'Academie Imperiale des Sciences de St.-Petersbourg*: 153-162.
- Markov Models and **Hidden Markov Models** are popular in
 - Language Modeling, Part-of-speech tagging
 - Speech recognition
 - Named entity recognition / information extraction
 - Biological sequence analysis

Markov Models

- Definition

*Assume an alphabet Σ . A **Markov Model** of order 1 is a sequential stochastic process with $|\Sigma|$ states s_1, \dots, s_n with*

- *Every state emits exactly one symbol from Σ*
- *No two states **emit the same symbol***
- *For a sequence $\langle w_1, w_2, \dots \rangle$ of states, the following holds*

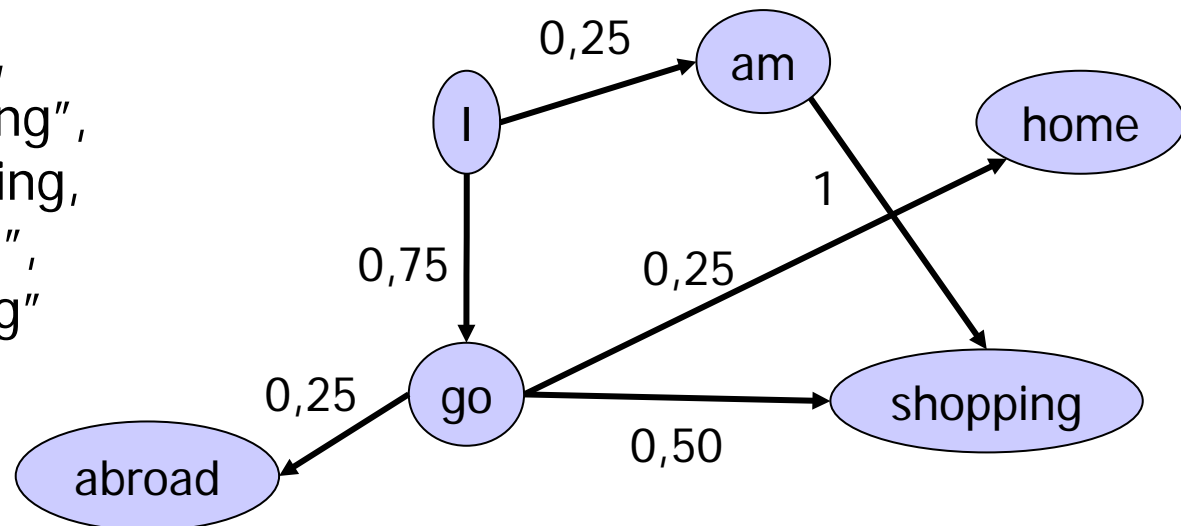
$$p(w_n=s_n/w_{n-1}=s_{n-1}, w_{n-2}=s_{n-2}, \dots, w_1=s_1) = p(w_n=s_n/w_{n-1}=s_{n-1})$$

- Remarks

- $a_{i,j} = p(w_n=s_j|w_{n-1}=s_i)$ are called **transition probabilities**
- In language modeling, $\Sigma = \{\text{set of all words of a language}\}$
- Computing good **start probabilities** is an issue we essentially ignore

Visualization

- Since every state emits exactly one word, we can merge states and words
- State transition graph
 - Nodes are states (labeled with their emission)
 - Arcs are transitions labeled with a non-zero probability
- Example
 - “I go home”,
 - “I go shopping”,
 - “I am shopping”,
 - “I go abroad”,
 - “Go shopping”

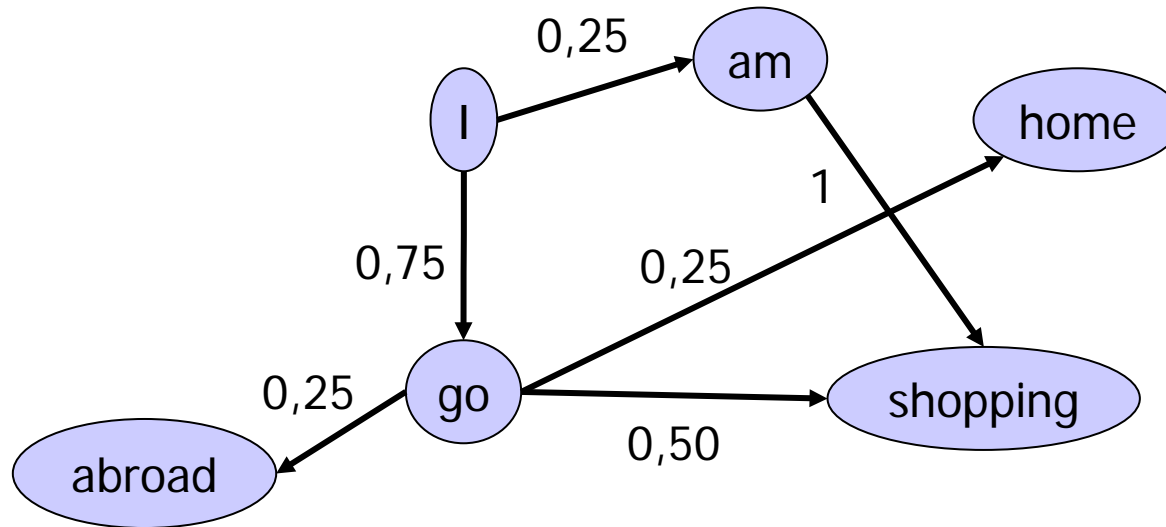


Probability of a Sequence of States (=a Sentence)

- Assume a Markov Model M of order 1 and a sequence S of states with $|S|=n$
- With which **probability was S generated by M** , i.e., what is the value of $p(S|M)$?

$$\begin{aligned} p(S | M) &= p(w_1 = S[1]) * \prod_{i=2..n} p(w_i = S[i] | w_{i-1} = S[i-1]) \\ &= a_{0,S[1]} * \prod_{i=2..n} a_{S[i-1],S[i]} = a_{0,1} * \prod_{i=2..n} a_{i-1,i} \end{aligned}$$

Example



- $p(\text{"I go home"}) = p(w_1 = \text{"I"} | w_0) * p(w_2 = \text{"go"} | w_1 = \text{"I"}) * p(w_3 = \text{"home"} | w_2 = \text{"go"})$
 $= 1 * 0.75 * 0.25 = 0.1875$
- Problem: **Pairs we have not seen** in training get prob. 0
 - Example: "I am abroad"
 - With this small "corpus", almost all transitions get $p=0$

Stochastic Processes

- Consider language generation as a **sequential stochastic process**
- At each stage, the process generates a new word
 - Like a DFA, but transitions have probabilities
- Question: How big is the **memory**? How many previous words does the process use to determine the next step?
 - 0: **Markov chain** of order 0: No memory at all
 - 1: Markov chain order 1: Next word only depends on prev. word
 - 2: Markov chain order 2: Next word only depends on 2 prev. words
 - ...

Higher Order Markov Models

- Markov Models of order k

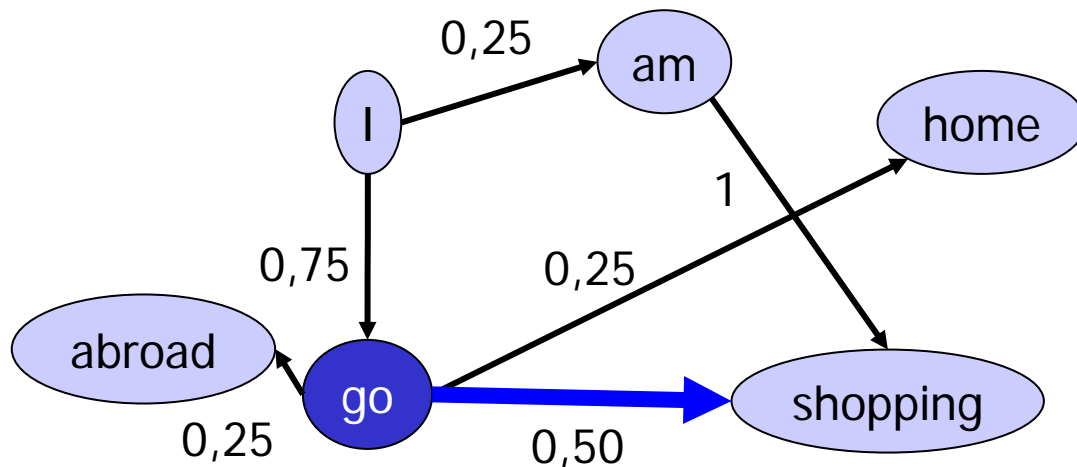
- The probability of being in state s after n steps depends on the k predecessor states s_{n-1}, \dots, s_{n-k}

$$p(w_n = s_n / w_{n-1} = s_{n-1}, w_{n-2} = s_{n-2}, \dots, w_1 = s_1) = p(w_n = s_n / w_{n-1} = s_{n-1}, \dots, w_{n-k} = s_{n-k})$$

- We can trivially transform any order k model M ($k > 1$) into a Markov Model of order 1 (M')
 - M' has $|M|^k$ states (all combinations of states of length k)

Predicting the Next State

- The problem of language modeling is a bit different
- We do not want to reason about an entire sequence, but only about the **next state**, given some previous states
- N-gram model = Markov Model order n-1



$$\begin{aligned} p(w_n) &= p(w_n | w_1, \dots, w_{n-1}) \\ &= p(w_n | w_{n-1}) \\ &= \frac{p(w_{n-1}, w_n)}{p(w_{n-1})} \\ &\sim p(w_{n-1}, w_n) \end{aligned}$$

This is the **most frequent bi-gram** with prefix w_{n-1}

Problem

- We learn our transition probabilities from **a limited sample**
- Thus, we only **estimate the true transition** probabilities
- Introduces an error which we should try to remove
 - **Sample selection** is important
 - Problem is researched a lot in statistics
- Extreme: Transitions we **do not see at all** in the corpus
 - Get a probability of 0
 - Will never be predicted
 - This does not mean that they are non-existing in the language
- Our model (yet) cannot adequately cope with **data sparsity**

Importance of Data Sparsity

- How many n-grams do exist in principle?
 - Assume a language of 20.000 words
 - $n=1$: 20.000, $n=2$: 4E8, $n=3$: 8E12, $n=4$: 1.6E17, ...
 - Rough numbers: Natural languages have many more words, but most combinations are not allowed
- In natural language corpora, almost all n-grams with $n > 4$ are very sparse
 - Exponential growth cannot be balanced by “use larger corpora”
 - Especially very specific n-grams are prone to be overlooked
- Trade-off
 - Large n : More expressive model, but bad transition estimations
 - Small n : Less expressive model, but better transition estimates

Example

- **Unigrams**: Always the most frequent word in the corpus, does not differentiate

<i>In</i>	<i>person</i>	<i>she</i>	<i>was</i>	<i>inferior</i>	<i>to</i>	<i>both</i>	<i>sisters</i>					
1-gram	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$					
1	<i>the</i>	0.034	<i>the</i>	0.034	<i>the</i>	0.034	<i>the</i>	0.034				
2	<i>to</i>	0.033	<i>to</i>	0.033	<i>to</i>	0.033	<i>to</i>	0.033				
3	<i>and</i>	0.030	<i>and</i>	0.030	<i>and</i>	0.030	<i>and</i>	0.030				
4	<i>of</i>	0.029	<i>of</i>	0.029	<i>of</i>	0.029	<i>of</i>	0.029				
...												
8	<i>was</i>	0.015	<i>was</i>	0.015	<i>was</i>	0.015	<i>was</i>	0.015				
...												
13	<i>she</i>	0.011		<i>she</i>	0.011	<i>she</i>	0.011	<i>she</i>	0.011			
...												
254				<i>both</i>	0.0005	both	0.0005	<i>both</i>	0.0005			
...												
435				<i>sisters</i>	0.0003		sisters	0.0003				
...												
1701				inferior	0.00005							
2-gram	$P(\cdot person)$	$P(\cdot she)$	$P(\cdot was)$	$P(\cdot inferior)$	$P(\cdot to)$	$P(\cdot both)$						
1	<i>and</i>	0.099	<i>had</i>	0.141	<i>not</i>	0.065	to	0.212	<i>be</i>	0.111	<i>of</i>	0.066
2	<i>who</i>	0.099	<i>was</i>	0.122	<i>a</i>	0.052			<i>the</i>	0.057	<i>to</i>	0.041
3	<i>to</i>	0.076			<i>the</i>	0.033			<i>her</i>	0.048	<i>in</i>	0.038
4	<i>in</i>	0.045			<i>to</i>	0.031			<i>have</i>	0.027	<i>and</i>	0.025
...												
23	she	0.009							<i>Mrs</i>	0.006	<i>she</i>	0.009
...												
41									<i>what</i>	0.004	sisters	0.006
...												
293									both	0.0004		
...												
∞									inferior	0		
3-gram	$P(\cdot In, person)$	$P(\cdot person, she)$	$P(\cdot she, was)$	$P(\cdot was, inf.)$	$P(\cdot inferior, to)$	$P(\cdot to, both)$						
1	UNSEEN	<i>did</i>	0.5	<i>not</i>	0.057	UNSEEN	<i>the</i>	0.286	<i>to</i>	0.222		
2		<i>was</i>	0.5	<i>very</i>	0.038		<i>Maria</i>	0.143	<i>Chapter</i>	0.111		
3				<i>in</i>	0.030		<i>cherries</i>	0.143	<i>Hour</i>	0.111		
4				<i>to</i>	0.026		<i>her</i>	0.143	<i>Twice</i>	0.111		
...												
∞				inferior	0		both	0	sisters	0		
4-gram	$P(\cdot u, l, p)$	$P(\cdot l, p, s)$	$P(\cdot p, s, w)$	$P(\cdot s, w, l)$	$P(\cdot w, l, t)$	$P(\cdot t, l, b)$						
1	UNSEEN	UNSEEN	<i>in</i>	1.0	UNSEEN	UNSEEN	UNSEEN					
...												
∞			inferior	0								

Table 6.3 Probabilities of each successive word for a clause from *Persuasion*. The probability distribution for the following word is calculated by Maximum Likelihood Estimate n -gram models for various values of n . The predicted likelihood rank of different words is shown in the first column. The actual next word is shown at the top of the table in italics, and in the table in bold.

Example

- Bi-grams: Correct word often ranks high, but not always

<i>In</i>	person	she	was	<i>inferior</i>	to	both	sisters			
1-gram	$P(\cdot)$		$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$			
1	the	0.034	the	0.034	the	0.034	the	0.034		
2	to	0.032	to	0.032	to	0.032	to	0.032		
3	and	0.030	and	0.030	and	0.030	and	0.030		
4	of	0.029	of	0.029	of	0.029	of	0.029		
...										
8	was	0.015	was	0.015	was	0.015	was	0.015		
...										
13	she	0.011			she	0.011	she	0.011		
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2-gram	$P(\cdot person)$	$P(\cdot she)$	$P(\cdot was)$	$P(\cdot inferior)$	$P(\cdot to)$	$P(\cdot both)$				
1	and	0.09	not	0.065	to	0.212	be	0.111	of	0.066
2	who	0.09	was	0.122	a	0.052	the	0.057	to	0.041
3	to	0.075			the	0.033	her	0.048	in	0.038
...					to	0.031	have	0.027	and	0.025
...										
2	she	0.009					Mrs	0.006	she	0.009
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∞					<i>inferior</i>	0				
3-gram	$P(\cdot In, person)$	$P(\cdot person, she)$	$P(\cdot she, was)$	$P(\cdot was, inf.)$	$P(\cdot inferior, to)$	$P(\cdot to, both)$				
1	UNSEEN	did	0.5	not	0.057	UNSEEN	the	0.286	to	0.222
2		was	0.5	very	0.038		Maria	0.143	Chapter	0.111
3				in	0.030		cherries	0.143	Hour	0.111
4				to	0.026		her	0.143	Twice	0.111
...										
∞				<i>inferior</i>	0		both	0	sisters	0
4-gram	$P(\cdot u, l, p)$	$P(\cdot l, p, s)$	$P(\cdot p, s, w)$	$P(\cdot s, w, l)$	$P(\cdot w, l, l)$	$P(\cdot l, l, b)$				
1	UNSEEN	UNSEEN	in	1.0	UNSEEN	UNSEEN	UNSEEN			
...										
∞			<i>inferior</i>	0						

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Example

<i>In</i>												
person	she	was	inferior	to	both	sisters						
1-gram	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$	$P(\cdot)$
1	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034
2	to	0.032	to	0.032	to	0.032	to	0.032	to	0.032	to	0.032
3	and	0.030	and	0.030	and	0.030	and	0.030	and	0.030	and	0.030
4	of	0.029	of	0.029	of	0.029	of	0.029	of	0.029	of	0.029
...												
8	was	0.015	was	0.015	was	0.015	was	0.015	was	0.015	was	0.015
...												
13	she	0.011			she	0.011			she	0.011	she	0.011
...												
254					both	0.0005			both	0.0005	both	0.0005
...												
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2-gram	$P(\cdot person)$	$P(\cdot she)$	$P(\cdot was)$	$P(\cdot inferior)$	$P(\cdot to)$	$P(\cdot both)$						
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3-gram	$P(\cdot in, person)$	$P(\cdot person, she)$	$P(\cdot she, was)$	$P(\cdot was, inf)$	$P(\cdot inferior, to)$	$P(\cdot to, both)$						
1	UNSEEN	did	0.5	not	0.057	UNSEEN	the	0.286	to	0.222		
2		was	0.5	very	0.038		Maria	0.143	Chapter	0.111		
3				in	0.030		cherries	0.143	Hour	0.111		
4				to	0.026		her	0.143	Twice	0.111		
...												
∞				inferior	0		both	0	sisters	0		
4-gram	$P(\cdot in, p)$	$P(\cdot p, s)$	$P(\cdot p, w)$	$P(\cdot w, d)$	$P(\cdot w, ll)$	$P(\cdot ll, b)$						
1	UNSEEN	UNSEEN	in	1.0	UNSEEN	UNSEEN						
...												
∞				inferior	0							

- Tri-grams: Has a hit, but already suffers from sparsity
- Four-grams: Unusable
- Corpus: Fraction of Jane Austen's oeuvre, ~600.000 tokens, data from [MS99]

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Solutions we will not Discuss in Detail

- Reduce the number of words using **stemming**
 - Might help to go from $n=3..4$ to $n=4..5$
 - Important grammatical clues are lost
- More abstract: Use some form of “binning” of **tokens into classes** and compute n-grams over token classes, not token
 - All numbers -> one class
 - All verbs -> one class (POS tags)
 - All verbs related to “movement” -> one class

Statistical Estimators

- We were a bit sloppy so far
- We want

$$p(w_n) = p(w_n | w_1, \dots, w_{n-1}) = \frac{p(w_1, \dots, w_n)}{p(w_1, \dots, w_{n-1})}$$

- But we only have $count(w_1, \dots, w_n)$
- So far, we always **implicitly assumed**

$$p(w_1, \dots, w_n) = \frac{count(w_1, \dots, w_n)}{N}$$

- N: all observed n-grams

MLE for N-gram Models

- This is called a **Maximum Likelihood Estimator** (MLE)
- MLE gives **maximum likelihood** to the training data
 - Gives zero probability to all events not in the training data
 - The **probability mass** is spent entirely on the training data
 - Overfitting
- Need to **smooth** the estimates to account for the limitations of the sample

Smoothing I: Laplace's Law

- Give some **probability mass to unseen events**
- Oldest (and simplest) suggestion: "Adding one"

$$p_{LAP}(w_1, \dots, w_n) = \frac{\text{count}(w_1, \dots, w_n) + 1}{N + B}$$

- Where B is the number of possible n-grams, i.e., K^n
 - K: Vocabulary, all different words
- All n-grams get a probability $\neq 0$
- But – **moves too much mass** to the unknown
 - Estimates for seen n-grams are scaled down dramatically
 - Estimates for unseen n-grams are small, but there are so many
 - And many of them are truly impossible
 - In a corpus of 40 M words with $K \sim 400T$, **99.7% of the total probability mass** is spend in unseen events

Smoothing II: Lidstone's Law

- Laplace not suitable if there are **many events, but few seen**
- Lidstone's law gives less probability mass to unseen events

$$p_{LIP}(w_1, \dots, w_n) = \frac{\text{count}(w_1, \dots, w_n) + \lambda}{N + \lambda * B}$$

- Small λ : More mass is given to seen events
- Typical estimate is $\lambda=0.5$
- Appropriate values can be **learned** (next slide)
- Still: Estimate of seen events is **linear in the MLE estimate**
 - Not a good approximation of empirical distributions
- Other: Good-Turing Estimator, n-gram interpolations, ...

Learning Appropriate Values for λ

- We “simulate” seen and unseen events
- Divide corpus in two disjoint parts C_1 and C_2
- Count frequencies of n-grams in C_1
- Let c be the number of n-grams from C_1 not present in C_2
- Set $\lambda=c/B$
 - The probability of an n-gram (in C_2) to be considered as not existing although in reality it does exist

Option III: Back-Off Models

- If we cannot find a n-gram with count≠0, use a (n-1)-gram
 - Or an n-2 gram, ...
- Thus, in case there is no $p(w_1, \dots, w_n) \neq 0$, we “back off” to a simpler model

$$p(w_n | w_1, \dots, w_{n-1}) = \frac{p(w_1, \dots, w_n)}{p(w_1, \dots, w_{n-1})} \text{ or } \frac{p(w_2, \dots, w_n)}{p(w_2, \dots, w_{n-1})} \text{ or } \frac{p(w_3, \dots, w_n)}{p(w_3, \dots, w_{n-1})} \text{ or } \dots$$

- Stop at the first (n-k)-gram with non-zero count
- Alternative: Always look at different n's
 - With different weights

$$p(w_n) = \lambda_1 \frac{p(w_{n-2}, w_{n-1}, w_n)}{p(w_{n-2}, w_{n-1})} + \lambda_2 \frac{p(w_{n-1}, w_n)}{p(w_{n-1})} + \lambda_3 p(w_n)$$

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New IR Model

- Recent trend in IR: Relevance based on language models
- Idea: See a **document as a “language”**
 - Learn a model M_d of this “language” (document)
 - Compute with which probability $p(d|q)$ a given **query has generated the model (=document)**
 - Rank documents based on these probabilities
- Sounds weird, but leads to a simple and well justified approach
- Very successful in recent evaluations
- **Smoothing is crucial** – docs are too small

Approach

- As docs are small, only **unigram models** are sensible
- Model of a doc: Relative frequencies of all its words
- Compute

$$p(d | q) = \frac{p(q | d) * p(d)}{p(q)} \sim p(q | d) * p(d) \sim p(q | d)$$

- $p(q)$ is equal for all d – irrelevant for ranking
- $p(d)$ can be used to incorporate a-prior knowledge (e.g. prestige), but often is set to uniform – irrelevant for ranking
- We **replace d with its model** and obtain

$$p(q | d) = p(q | M_d) = p(k_1, k_2, \dots, k_n | M_d) = \prod_{k \in q} p(k | M_d) = \prod_{k \in q} \frac{tf_{k,d}}{|d|}$$

Discussion

- **Very simple**
- Principled approach to justify usage of tf values
- More powerful for longer queries
- Problems
 - Words in q not in d: **Smoothing**
 - Where is idf gone?

Smoothing a Language Model for IR

- For instance, if $k \notin d$, set $p(k|M_d) = df_k/|D| = p(k|M_D)$
 - Token that are in d are counted with tf values (and not discounted with idf); tokens **not in d are counted with df values**
- More tunable parameters: **Linear interpolation**

$$p'(k | M_d) = \lambda * p(k | M_d) + (1 - \lambda) * p(k | M_D)$$

- Combine **relevance of k in document** and **relevance of k in corpus**
- Large λ : More weight to the document, less weight to background
- λ may vary, for instance with query size
- We are back at something similar to TF*IDF, but with a **probabilistic interpretation**, not a geometric one

Self Assessment

- What is language modelling about?
- Define a Markov model
- How can you turn a Markov model of order 4 into one of order 1?
- What is the data sparsity problem (in language modeling)?
- What is the disadvantage of Laplace smoothing?
- Explain how we can use language models for information retrieval