HUMBOLDT-UNIVERSITÄT ZU BERLIN

Information Retrieval

Searching Terms

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Content of this Lecture

- Searching strings
- Naïve exact string matching
- Boyer-Moore
- BM-Variants and comparisons

Searching Strings in Text

- All IR models require finding occurrences of terms in documents
- Fundamental operation: find(k,D) -> $\mathcal{P}(D)$
- Indexing: Preprocess docs and use index for searching
  - Apply tokenization; can only find entire words
  - Classical IR technique (inverted files)
- Online searching: Consider docs and query as new
  - No preprocessing - slower
  - Usually without tokenization – more “searchable” substrings
  - Classical algorithmic problem: Substring search

Properties

- Advantages of substring search
  - Does not require (erroneous, ad-hoc) tokenization
    - “U.S.”, “35,00=.000”, “alpha-type1 AML-3’ protein”, ...
  - Search across tokens / sentences / paragraphs
    - “, that ”, “happen. “, ...
  - Searching prefixes, infixes, suffixes, stems
    - “compar”, “ver” (German), ...
- Searching substrings is “harder” than searching terms
  - Number of unique terms doesn’t increase much with corpus size (from a certain point on)
    - English: ~ 1 Million terms, but 200 Million potential substrings of size 6
  - Need to index all possible substrings

Types of Substring Searching

- Exact search: Find all exact occurrences of a pattern (substring) p in D
- RegExp matching: Find all matches of a regular exp. p in D
- Approximate search: Find all substrings in D that are “similar” to a pattern p
  - Phonetically similar (Soundex)
  - Only one typo away (keyboard errors)
  - Strings that can be produced from p by at most n operations of type “insert a letter”, “delete a letter”, “change a letter”
  - ...
- Multiple strings: Searching >1 strings at once in D

Definition: Strings

- A *String S is a sequential, ordered list of characters from a finite alphabet* Σ
  - *|S| is the number of characters in S*
  - *Positions in S are counted from 1,…,|S|*
  - *S[i] denotes the character at position i in S*
  - *S[i..j] denotes the substring of S starting at position i and ending at position j (including both)*
  - *S[1..i] or S[..i] is the prefix of S until position i*
  - *S[i..|S|] or S[i..] is the suffix of S starting from position i*
  - *S[..i] (S[i..]) is called a proper prefix (suffix) of S iff*
    - *i≠0 (not empty) and*
    - *i≠|S| (not entire String).*

```
   123456789…
S  dadfabzzb…
```

Exact Substring Matching

- Given: Pattern P to search for, text T to search in
  - We require |P| ≤ |T| (T is longer than P)
  - We assume |P| << |T| (T is much longer than P)
- Task: Find all occurrences of P in T
  - Where is “GATATC”

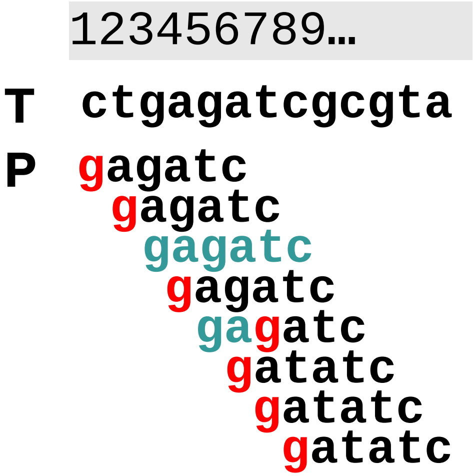
```
tcagcttactaattaaaaattctttctagtaagtgctaagatcaagaaaataaattaaaaataatggaacatggcacattttcctaaactcttcacagattgctaatga
ttattaattaaagaataaatgttataatttttatggtaacggaatttcctaaaatattaattcaagcaccatggaatgcaaataagaaggactctgttaattggtact
attcaactcaatgcaagtggaactaagttggtattaatactctttttacatatatatgtagttattttaggaagcgaaggacaatttcatctgctaataaagggattac
atatttatttttgtgaatataaaaaatagaaagtatgttatcagattaaacttttgagaaaggtaagtatgaagtaaagctgtatactccagcaataagttcaaataggc
gaaaaacttttaataacaaagttaaataatcattttgggaattgaaatgtcaaagataattacttcacgataagtagttgaagatagtttaaatttttctttttgtatt
acttcaatgaaggtaacgcaacaagattagagtatatatggccaataaggtttgctgtaggaaaattattctaaggagatacgcgagagggcttctcaaatttattcaga
gatggatgttttagatggtggtttaagaaaagcagtattaaatccagcaaaactagaccttaggtttattaaagcgaggcaataagttaattggaattgtaaaagatat
ctaattcttcttcatttgttggaggaaaactagttaacttcttaccccatgcagggccatagggtcgaatacgatctgtcactaagcaaaggaaaatgtgagtgtagact
ttaaaccatttttattaatgactttagagaatcatgcatttgatgttactttcttaacaatgtgaacatatttatgcgattaagatgagttatgaaaaaggcgaatatat
tattcagttacatagagattatagctggtctattcttagttataggacttttgacaagatagcttagaaaataagattatagagcttaataaaagagaacttcttggaat
tagctgcctttggtgcagctgtaatggctattggtatggctccagcttactggttaggttttaatagaaaaattccccatgattgctaattatatctatcctattgagaa
caacgtgcgaagatgagtggcaaattggttcattattaactgctggtgctatagtagttatccttagaaagatatataaatctgataaagcaaaatcctggggaaaatat
tgctaactggtgctggtagggtttggggattggattatttcctctacaagaaatttggtgtttactgatatccttataaataatagagaaaaaattaataaagatgatat
```

How to do it?

- The straight-forward way (naïve algorithm)
  - We use two counters: t, p
  - One (outer, t) runs through T
  - One (inner, p) runs through P

  - Compare characters at position T[t+p-1] and P[p]

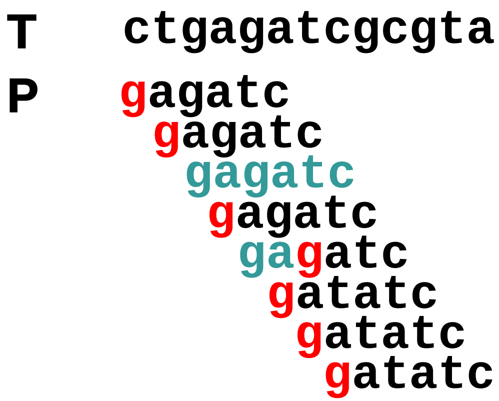
```
for t = 1 to |T| - |P| + 1
       p := 1;
       while (p <= |P| and T[t+p-1] == P[p])
              p := p + 1;
       end while;

       if (p == |P|) then
              REPORT t
end for;
```



```
   123456789…
T  ctgagatcgcgta
P  gagatc
    gagatc
     gagatc
      gagatc
       gagatc
        gatatc
         gatatc
          gatatc
```

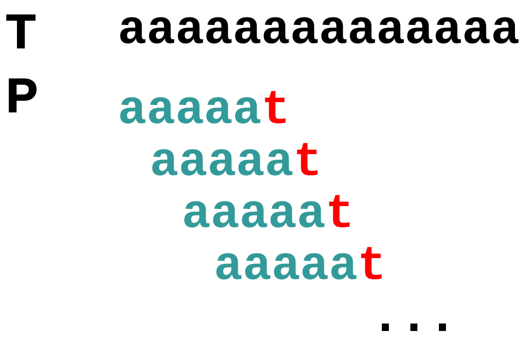
Examples

Typical case



```
T   ctgagatcgcgta
P   gagatc
     gagatc
      gagatc
       gagatc
        gagatc
         gatatc
          gatatc
           gatatc
```

Worst case



```
T   aaaaaaaaaaaaaa
P   aaaaat
     aaaaat
      aaaaat
       aaaaat
            ...
```

- How many comparisons do we need in the worst case?
  - t runs through T
  - p runs through the entire P for every value of t
  - Thus: |P|*|T| comparisons
  - Indeed: The algorithm has worst-case complexity O(|P|*|T|)

Other Algorithms

- Exact substring search has been researched for decades
  - Boyer-Moore, Z-Box, Knuth-Morris-Pratt, Karp-Rabin, Shift-AND, ...
  - All have WC complexity $O(|P| + |T|)$
  - For many, WC=AC, but empirical performance differs much
  - Real performance depends much on the size of alphabet and the composition of strings (algs have their strength in certain settings)
  - Better performance possible if T is preprocessed (up to O(|P|))
- In practice, our naïve algorithm is quite competitive for non-trivial alphabets and biased letter frequencies
  - E.g., English text
- But we can do better: Boyer-Moore
  - We present a simplified form
  - BM is among the fastest algorithms in practice

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- Searching strings
- Naïve exact string matching
- Boyer-Moore
- BM-Variants and comparisons

Boyer-Moore Algorithm

- R.S. Boyer /J.S. Moore. „A Fast String Searching Algorithm“, Communications of the ACM, 1977
- Main idea
  - As for the naïve alg, we use two counters (inner loop, outer loop)
  - Inner loop runs from right-to-left
  - If we reach a mismatch, we know
    - The character in T we just haven't seen
      - This is captured by the bad character rule
    - The suffix in P we just have seen
      - This is captured by the good suffix rule
- Use this knowledge to make longer shifts in T

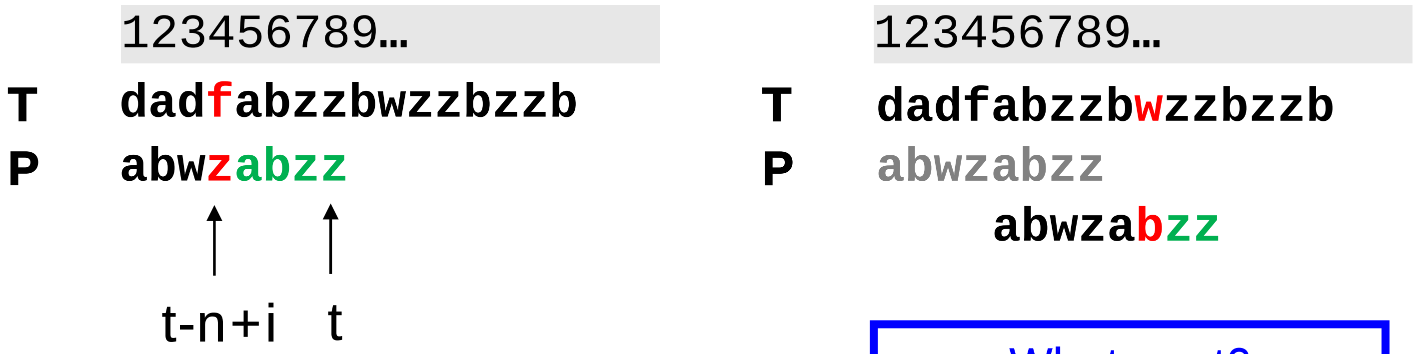
Boyer-Moore Main Idea

- Inner loop runs from right-to-left
- If we reach a mismatch, and this bad character does not appear in P at all, we can shift the pattern P my |P| positions:

```
     123456789…
T    dadfabzzbwzzbzzb
P    aaba
         aaba
```

Bad Character Rule

- Setting 1
  - We are at position *t* in *T* and compare right-to-left
  - Let *i* by the position of the first mismatch in *P*, *n*=|P|
    - We saw *n-i* matches before
  - Let *x* be the character at the corresponding pos *(t-n+i)* in *T*
  - Candidates for matching x in *P*
    - Case 1: x does not appear in P at all – we can move t such that t-n+i is not covered by P anymore



```
     123456789...
T    dadfabzzbwzzbzzb
P    abwzabzz
        ↑    ↑
    t-n+i    t
```

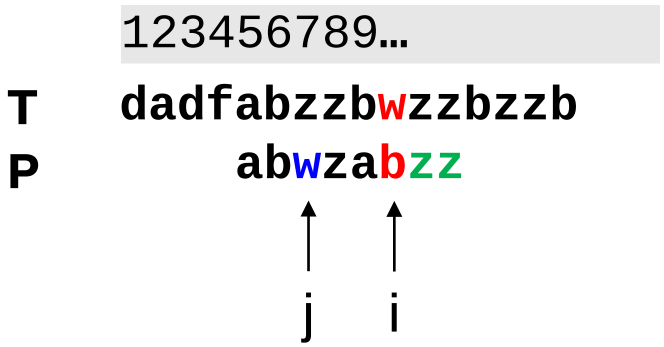
```
     123456789...
T    dadfabzzbwzzbzzb
P    abwzabzz
         abwzabzz
```

What next?

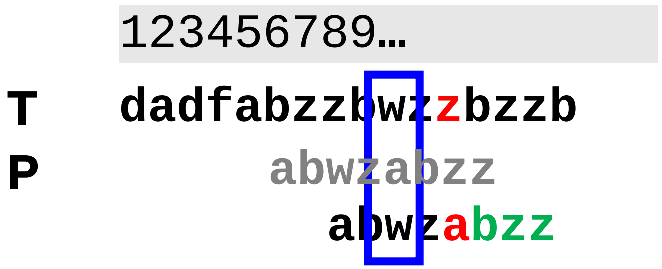
Bad Character Rule 2

	j
'a'	5
'b'	6
'w'	3
'z'	8

- Setting 2
  - We are at position *t* in *T* and compare right-to-left
  - Let *i* by the position of the first mismatch in *P, n=*|P|
  - Let *x* be the character at the corresponding pos *(t-n+i)* in *T*
  - Candidates for matching x in *P*
    - Case 1: x does not appear in P at all
    - Case 2: Let j be the right-most appearance of x in P and let j<i – we can move t such that j and t align



```
   123456789…
T  dadfabzzbwzzbzzb
P      abwzabzz
          ↑   ↑
          j   i
```

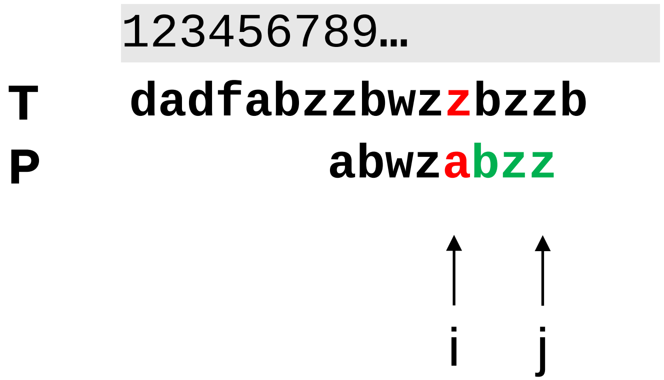


```
   123456789…
T  dadfabzzbwzzbzzb
P       abwzabzz
          abwzabzz
```

What next?

Bad Character Rule 3

- Setting 3
  - We are at position t in T and compare right-to-left
  - Let *i* by the position of the first mismatch in *P, n*=|P|
  - Let *x* be the character at the corresponding pos *(t-n+i)* in *T*
  - Candidates for matching x in *P*
    - Case 1: x does not appear in P at all
    - Case 2: Let j be the right-most appearance of x in P and let j<i
    - Case 3: As case 2, but j>i – we need some more knowledge



```
     123456789…
T    dadfabzzbwzzbzzb
P              abwzabzz
```

Preprocessing 1

- In case 3, there are some “x” right from position i
  - For small alphabets (DNA), this will almost always be the case
  - In human languages, this is often the case (e.g. for vowels)
  - Thus, case 3 is a usual one
- These “x” are irrelevant – we need the right-most x left of *i*
- This can (and should!) be pre-computed
  - Build a two-dimensional array *A[|Σ|,|P|]*
  - Run through *P* from left-to-right (pointer *i*)
  - If character *c* appears at position *i*, set all *A[c,j]:=i* for all *j>=i*
  - Requested time (complexity?) negligible
    - Because |P|<<|T| and complexity independent from T
- Array: Constant lookup, needs some space (lists …)

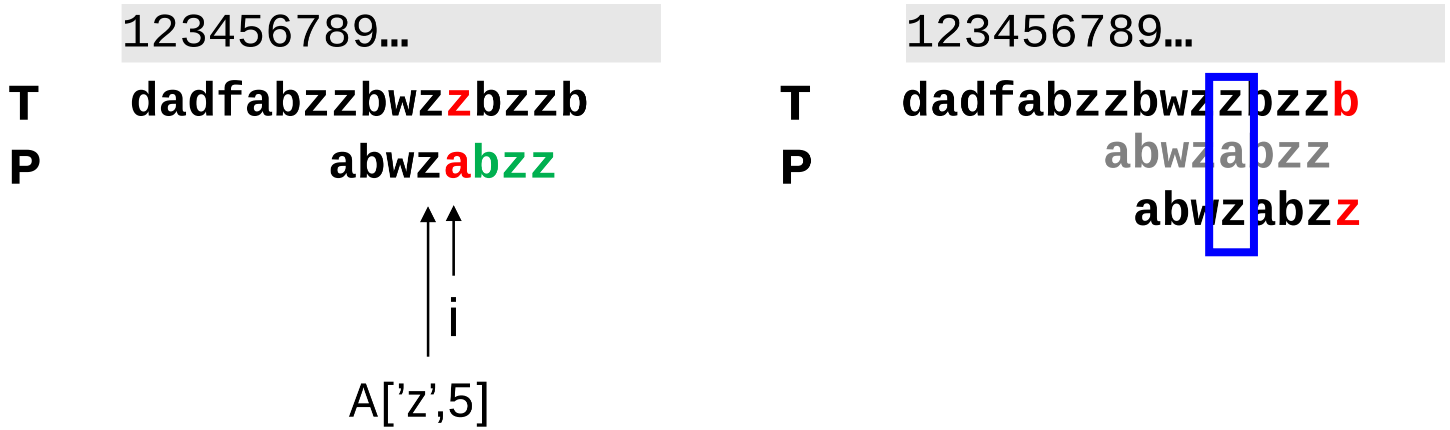
(Extended) Bad Character Rule

12345678

P abwzabzz

A

	1	2	3	4	5	6	7	8
‘a’	1	1	1	1	5	5	5	5
‘b’	0	2	2	2	2	6	6	6
‘w’	0	0	3	3	3	3	3	3
‘z’	0	0	0	4	4	4	7	8



(Extended) Bad Character Rule

- EBCR: Shift *t* by *i-A[x,i]* positions
- Simple and effective for larger alphabets
- For random strings over Σ, average shift-length is |Σ|/2
  - Thus, n# of comparisons down to |T|*2/|Σ|
- Worst-Case complexity does not change
  - Why?

(Extended) Bad Character Rule

- EBCR: Shift *t* by *i-A[x,i]* positions
- Simple and effective for larger alphabets
- For random strings over Σ, average shift-length is |Σ|/2
  - Thus, n# of comparisons down to |T|*2/|Σ|
- Worst-Case complexity does not change
  - Why? Shift-length can be always 1:

T=g^m

gggggggggggggggggggggggggggggggggggggggggg

P=ag^n

aggggggggggg

aggggggggggg

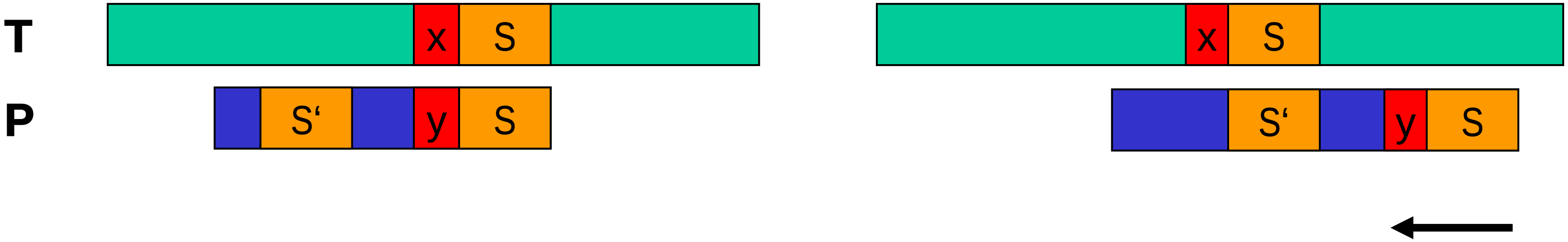
aggggggggggg

aggggggggggg

O(|P||T|)*

Good-Suffix Rule

- Recall: If we reach a mismatch, we know
  - The character in T we just haven’t seen
  - The suffix in P we just have seen
- Good suffix rule
  - We have just seen some matches (let these be S) in P
  - Where else does S appear in P?
  - If we know the right-most appearance S’ of S in P, we can immediately align S’ with the current match in T
  - If S does not appear anymore in P, we can shift t by |P|

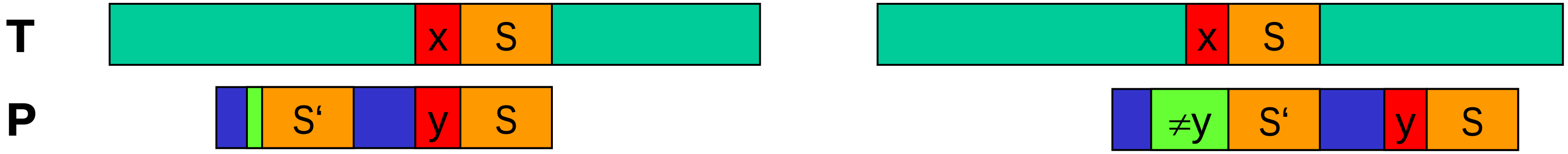


Good-Suffix Rule – One Improvement

- Actually, we can do a little better
- Not all S‘ are of interest to us

Good-Suffix Rule – One Improvement

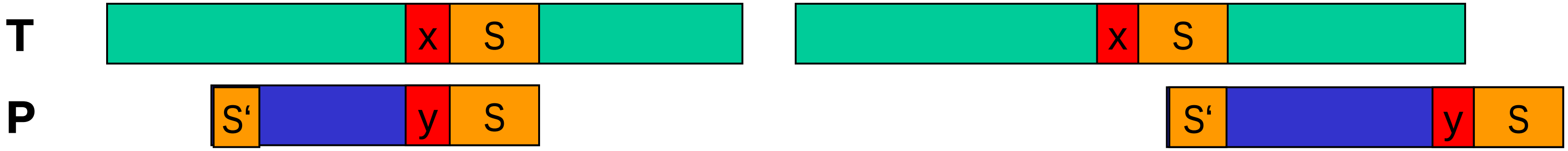
- Actually, we can do a little better
- Not all S‘ are of interest to us



- We only need S‘ whose next character to the left is not y
- Why don‘t we directly require that this character is x?
  - Of course, this could be used for further optimization

Good-Suffix Rule

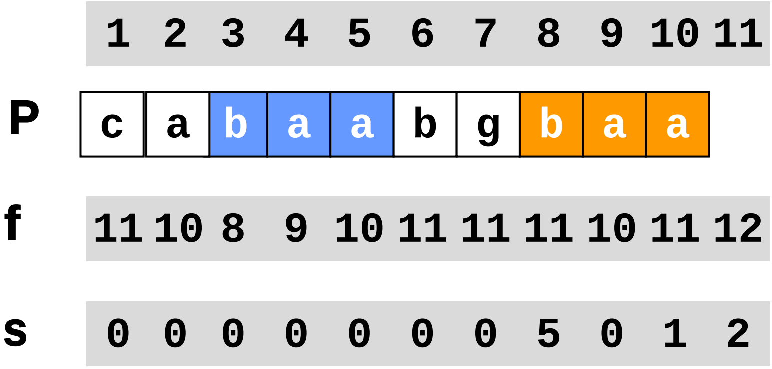
- Special case: Let S’ be a suffix of S and S’ be a prefix of P :



- We have to align S‘ with S.

Good-Suffix-Rule – Preprocessing 2

- Use two arrays:
  - Position of the longest suffix *f*: f[i] stores the starting position of prefix P[i..] in the suffix of P.
  - Maximum shift *s*: s[i] stores for position i the maximum shift to the left.



	1	2	3	4	5	6	7	8	9	10	11
P	c	a	b	a	a	b	g	b	a	a	
f	11	10	8	9	10	11	11	11	10	11	12
s	0	0	0	0	0	0	0	5	0	1	2

Concluding Remarks

- Preprocessing 2
  - For the GSR, we need to find all occurrences of all suffixes of P in P
  - This can be solved using our naïve algorithm for each suffix
  - Or, more complicated, in linear time (not this lecture)

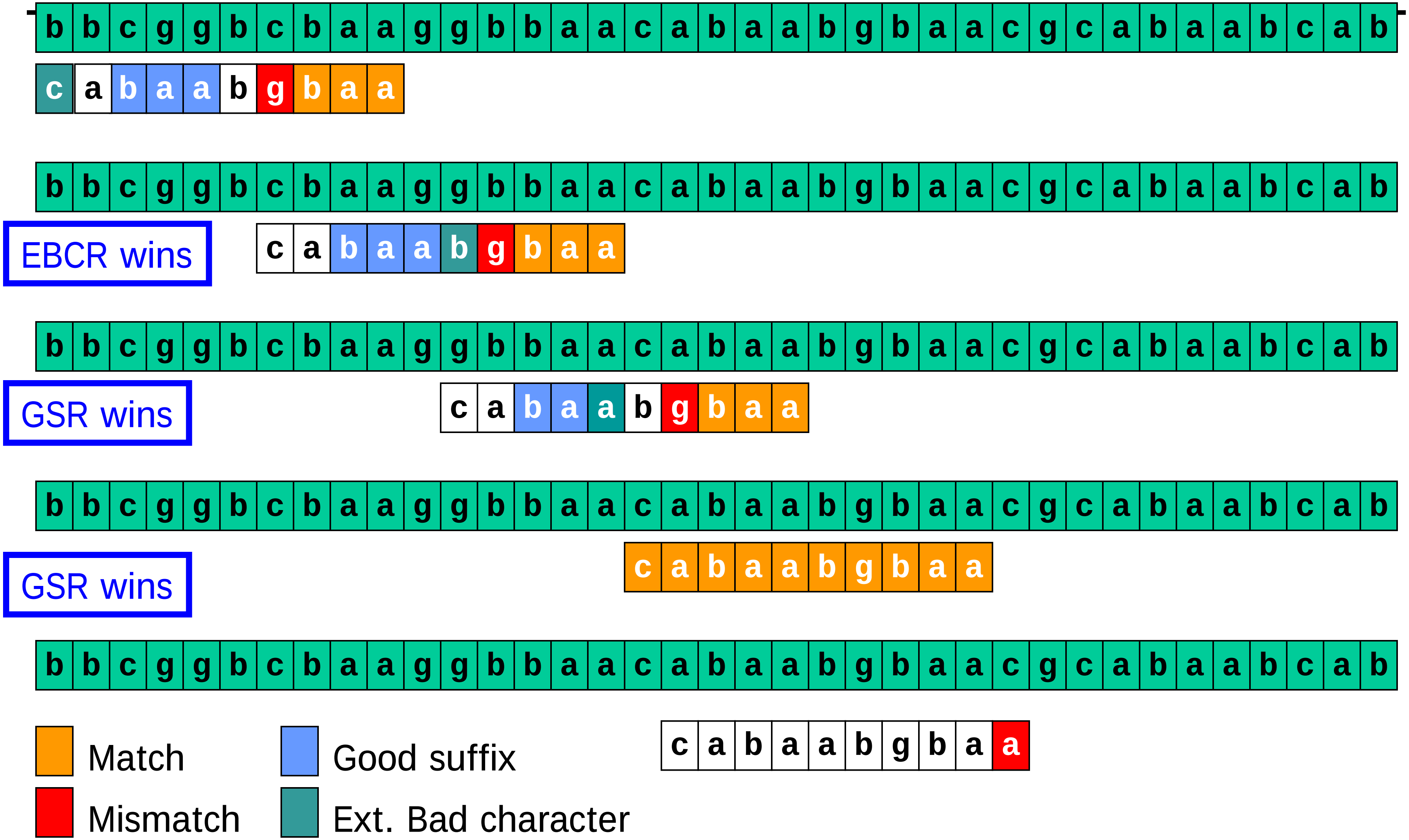
- WC complexity of Boyer-Moore is still O(|P|*|T|)
  - But average case is sub-linear: O(|T|/|P|); especially when |Σ|>|P|, which causes many shifts by |P|.
  - WC complexity can be reduced to linear (not this lecture), but this usually doesn't pay-off on real data

Boyer-Moore - Algorithm

- Compare characters at position T[t+|P|-p-1] and P[p]
  - t runs from left-to-right through T;
  - p runs from right-to-left through P;
- Mismatch: shift by maximum of GSR and EBCR.
- Match found: shift using GSR.

```
for t := 1 to |T|-|P|+1 do
        p := |P|;
        while (p > 0 and T[t+|P|-p-1] == P[p]) do
                p := p-1; end while
        if (p==0) then // match
                REPORT t;
                shift t to largest prefix of P, which is also a suffix of P
        else  // no match
                shift t by GSR, EBCR;
end for
```

Example



b b c g g b c b a a g g b b a a c a b a a b g b a a c g c a b a a b c a b

c a b a a b g b a a

b b c g g b c b a a g g b b a a c a b a a b g b a a c g c a b a a b c a b

EBCR wins

c a b a a b g b a a

b b c g g b c b a a g g b b a a c a b a a b g b a a c g c a b a a b c a b

GSR wins

c a b a a b g b a a

b b c g g b c b a a g g b b a a c a b a a b g b a a c g c a b a a b c a b

GSR wins

c a b a a b g b a a

b b c g g b c b a a g g b b a a c a b a a b g b a a c g c a b a a b c a b

c a b a a b g b a a

🟧 Match

🟥 Mismatch

🟦 Good suffix

🟩 Ext. Bad character

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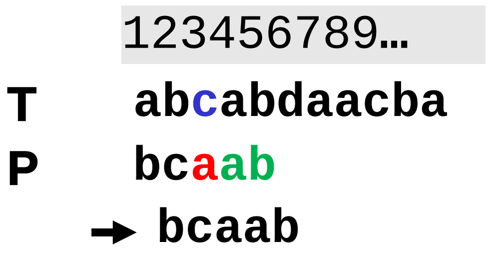
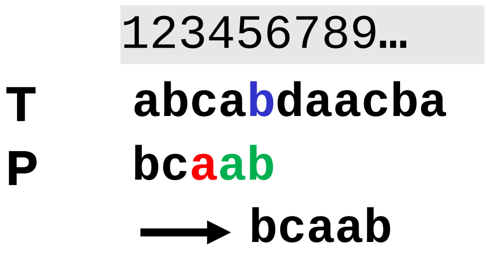
Two Faster Variants

- BM-Horspool
  - Drop the good suffix rule – GSR makes algorithm slower in practice
    - Rarely shifts longer than EBCR
    - Needs time to compute the shift
  - Instead of looking at the mismatch character x, always look at the symbol in T aligned to the last position of P
    - Generates longer shifts on average (i is maximal)
- BM-Sunday
  - Instead of looking at the mismatch character x, always look at the symbol in T after the symbol aligned to the last position of P
    - Generates even longer shifts on average
- Alternative: Always look at the least frequent (in the language of T) symbol of P first

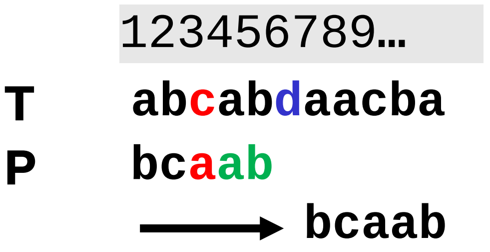
BM Variants



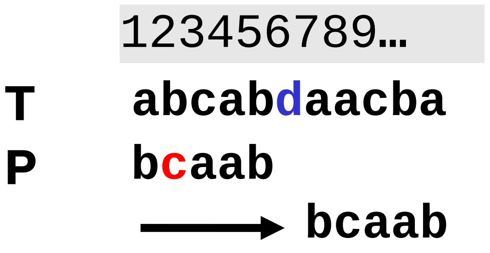
(a) Boyer-Moore



(b) BM-Horspool



(c) BM-Sunday

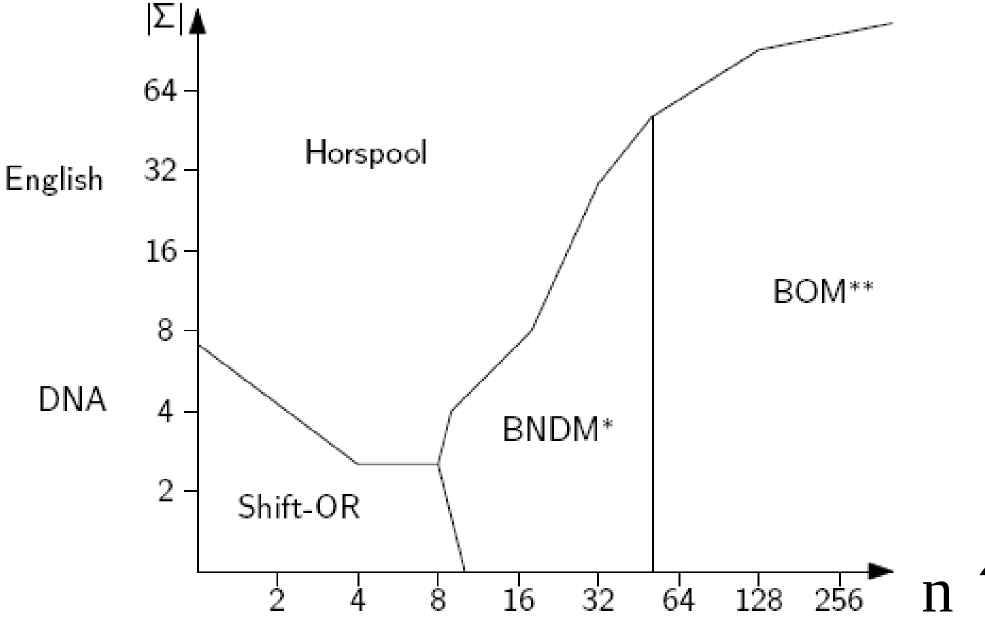


(d) BM-Sunday +

Least Frequent Char

Empirical Comparison

(Gonzalo Navarro & Mathieu Raffinot, 2002)



Machine word

- Shift-OR: Using parallelization in CPU (only small alphabets)
- BNDM: Backward nondeterministic Dawg Matching (automata-based)
- BOM: Backward Oracle Matching (automata-based)

Self Assessment

- Explain the Boyer-Moore algorithm
- Which rule is better – GSR or EBCR?
- How can we efficiently implement EBCR?
- How does the Sunday algorithm deviate from BM?
- How can we use character frequencies to speed up BM? If we do so - which part of the algorithm is sped up?