Information Retrieval

Modeling Information Retrieval 2

Ulf Leser

Content of this Lecture

- IR Models
- Boolean Model
- Vector Space Model
- Relevance Feedback in the VSM
- Probabilistic Model
- Latent Semantic Indexing
- Other IR Models

A Probabilistic Interpretation of Relevance

- VSM is fairly heuristic – some kind of similarity with some kind of weighting in some kind of vector space
- Probabilistic models build on well-established and mathematically consistent probability theory
  - Derive relevance formulas from a few basic and sound principles
- Probabilistic model
  - Words appearing in docs are seen as independent events
  - A doc (or query) is a conjunction of events
  - Compute the probability that a doc d is relevant to query q
    - Actually, we will compute a score (using probabilities)

Process

- We represent d as the set $\{k_i\}$ of words contained in d
  - Frequency of words not considered
- Initial queries too short for probabilistic reasoning
  - We need relevant docs
  - Determined iteratively using feedback (automatic, explicit, implicit)
    - Similar to VSM with relevance feedback
- Process for answering q
  - Subset $R \subseteq D$ of only relevant documents
  - Subset $N \subseteq D$ of only irrelevant documents
  - Compute $p(R|d)$, the probability that document d belongs to R
  - Typically performed iteratively

Odds-Score

- Let rel(d,q) be the relevance of a document comprising the terms of d for being relevant to query q
- Since words k_i appear both in relevant and in irrelevant docs, we look at the probability of both classes R / N
  - Also called odds-score

$$rel(d,q) = \frac{p(R \mid d)}{p(N \mid d)} = \frac{p(R \mid k_1,\dots,k_n)}{p(N \mid k_1,\dots,k_n)}$$

- Assuming statistical independence of words, we get

$$rel(d,q) = \frac{p(R \mid k_1,\dots,k_n)}{p(N \mid k_1,\dots,k_n)} = \frac{p(R \mid k_1) * \dots * p(R \mid k_n)}{p(N \mid k_1) * \dots * p(N \mid k_n)}$$

Using Bayes

- Using Bayes Theorem

$$rel(d,q) = \frac{p(R|d)}{p(N|d)} = \frac{p(d|R)*p(R)*p(d)}{p(d|N)*p(N)*p(d)} \sim \frac{p(d|R)}{p(d|N)}$$

- p(R/N): relative frequency of (ir-)relevant docs in D
  - A-Priori probability of a doc to be (ir-)relevant
  - Constant for a given q and thus irrelevant for ranking docs
- p(d|R) is the probability of drawing the combination of words forming d when drawing words at random from R

Binary Independence Model

- More intuitive: p(d|R) is the probability of drawing words from d from R and not drawing words not in d from R
- Binary Independence Model

$$rel(d,q) = \frac{p(d \mid R)}{p(d \mid N)} = \frac{\prod_{k \in d} p(k \mid R) \; * \prod_{k \notin d} p(\neg k \mid R)}{\prod_{k \in d} p(k \mid N) \; * \prod_{k \notin d} p(\neg k \mid N)}$$

  – Having words that are frequent in R raises the relevance of d
  – Not having words that are frequent in R lowers the relevance of d
  – Having words that are frequent in N lowers the relevance of d
  – Not having words that are frequent in N raises the relevance of d

Binary Independence Model

Good: Important terms for R present?

Bad: Important terms for R missing?

$$rel(d,q) = \frac{\prod_{k \in d} p(k|R) \; * \; \prod_{k \notin d} p(\neg k|R)}{\prod_{k \in d} p(k|N) \; * \; \prod_{k \notin d} p(\neg k|N)} = \frac{\prod_{k \in d} p(k|R) \; * \; \prod_{k \notin d} \left(1 - p(k|R)\right)}{\prod_{k \in d} p(k|N) \; * \; \prod_{k \notin d} \left(1 - p(k|N)\right)}$$

Bad: Important terms for N present?

Good: Important terms for N missing?

Continuation

- Rephrasing using q

$$rel(d,q) = \frac{\prod_{k \in d \cap q} p(k|R)}{\prod_{k \in d \cap q} p(k|N)} * \cancel{\frac{\prod_{k \in d \setminus q} p(k|R)}{\prod_{k \in d \setminus q} p(k|N)}} * \frac{\prod_{k \in q \setminus d} p(\neg k|R)}{\prod_{k \in q \setminus d} p(\neg k|N)} * \cancel{\frac{\prod_{k \notin d \cup q} p(\neg k|R)}{\prod_{k \notin d \cup q} p(\neg k|N)}}$$

- Since we are not sure about R/N: Focus on query terms

$$\ldots \approx \prod_{k \in d \cap q} \frac{p(k|R)}{p(k|N)} * \prod_{k \in q \setminus d} \frac{p(\neg k|R)}{p(\neg k|N)} = \prod_{k \in d \cap q} \frac{p(k|R)}{p(k|N)} * \prod_{k \in q \setminus d} \frac{1-p(k|R)}{1-p(k|N)}$$

Last Step

$$\prod_{k \in d \cap q} \frac{p(k \mid R)}{p(k \mid N)} * \prod_{k \in q \setminus d} \frac{1 - p(k \mid R)}{1 - p(k \mid N)}$$

All matching terms

All non-matching terms

- Some reformulating (duplicating the terms in q)

$$= \prod_{k \in d \cap q} \frac{p(k \mid R) * (1 - p(k \mid N)) * (1 - p(k \mid R))}{p(k \mid N) * (1 - p(k \mid R)) * (1 - p(k \mid N))} * \prod_{k \in q \setminus d} \frac{1 - p(k \mid R)}{1 - p(k \mid N)}$$

$$= \prod_{k \in d \cap q} \frac{p(k \mid R) * (1 - p(k \mid N))}{p(k \mid N) * (1 - p(k \mid R))} * \prod_{k \in q} \frac{1 - p(k \mid R)}{1 - p(k \mid N)}$$

All matching terms

All query terms

Problem

- Last term is identical for all d and can be dropped

$$rel(d,q) \approx \prod_{k \in d \cap q} \frac{p(k|R) * (1 - p(k|N))}{p(k|N) * (1 - p(k|R))}$$

- But: Computing rel(d,q) requires knowledge of R and N
  - If R and N were known, we could much easier estimate p(k|R) / p(k|N) by looking at the relative frequencies of terms in R/N
  - Also called maximum likelihood estimation
- In reality, we actually want to find R and N

Back to Reality

- Idea: Approximation using an iterative process
  - Start with “educated guess” for R and set N=D\R
    - E.g. R ~ “all docs containing at least one word from q”
  - Compute relevance of all docs with respect to q
  - Chose relevant docs (by user feedback) or hopefully relevant docs (by selecting the top-r docs)
  - This gives new sets R and N
    - If top-r docs are chosen, we may decide to only change probabilities of terms in R (and disregard the questionable negative information)
  - Compute new conditional probabilities and new ranking
  - Iterate until satisfied
- [Variant of the Expectation Maximization Algorithm (EM)]

Initialization

$$rel(d,q) \approx \prod_{k \in d \cap q} \frac{p(k|R)*(1-p(k|N))}{p(k|N)*(1-p(k|R))}$$

- Typical simplifying assumptions for the start
  - Terms in non-relevant docs are equally distributed: $p(k|N) \sim df_k/|D|$
  - Terms in relevant doc get equal probability: $p(k|R)=0.5$
  - Much less computation, less weight to unstable first values
  - [But leaves axiomatic probability theory]
- Iterations: Assume we have a new R' and N'. Then:

$$P(k|R') = \frac{|\{d \mid k \in d, d \in R'\}|}{|R'|}$$

$$P(k|N') = \frac{df_k - |\{d \mid k \in d, d \in R'\}|}{|D| - |R'|}$$

Example

	Text	verkauf	haus	italien	gart	miet	blüh	woll
1	Wir verkaufen Häuser in Italien	1	1	1				
2	Häuser mit Gärten zu vermieten		1		1	1		
3	Häuser: In Italien, um Italien, um Italien herum		1	1				
4	Die italienschen Gärtner sind im Garten			1	1			
5	Um unser italiensches Haus blüht‘s		1	1			1	
6	Wir verkaufen Blühendes	1					1	
Q	Wir wollen ein Haus mit Garten in Italien mieten		1	1	1	1		1

Example: Initialization

$$rel(d,q) \approx \prod_{k \in d \cap q} \frac{p(k|R) * (1 - p(k|N))}{p(k|N) * (1 - p(k|R))}$$

	V	H	I	G	M	B	W
1	1	1	1				
2		1		1	1		
3		1	1				
4			1	1			
5		1	1			1	
6	1					1	
Q		1	1	1	1		1

- All docs with at least one word from q
  - R={1,2,3,4,5}, N={6}
- Initial estimations
  - p(k|R)=0.5, p(k|N)= df_k/|D| -> p(verkauf|N)=p(blüh|N)=2/6
  - Smoothing: If p(k|X)=0, set p(k|X)=0.01
- Initial ranking
  - rel(1,q)= p(haus|R)*(1-p(haus|N))*p(italien|R)*(1-p(italien|N)) /
    p(haus|N)*(1-p(haus|R))*p(italien|N)*(1-p(italien|R))
    = .5*(1-0.01)*.5*(1-0.01) / (0.01*(1-0.5)*0.01*(1-0.5))=
    = 9801
  - rel(2,q)= 970299
  - rel(3,q)= rel(4,q) = rel(5,q) = 9801
  - rel(6,q)= 0

Adjustment

$$P(k \mid R) = \frac{|\{d \mid k \in d, d \in R\}|}{|R|}$$

$$P(k \mid N) = \frac{df_k - |\{d \mid k \in d, d \in R\}|}{|D| - |R|}$$

	V	H	I	G	M	B	W
1	1	1	1				
2		1		1	1		
3		1	1				
4			1	1			
5		1	1			1	
6	1					1	
Q		1	1	1	1		1

- Let's use the top-2 docs as new R
  - Second chosen arbitrarily among 1,3,4,5
  - R={1,2}, N={3,4,5,6}
- Adjust scores
  - p(verkauf|R)=.5, p(verkauf|N)=(2-1)/(6-2)=1/4
  - p(haus|R)=1 (~.99), p(haus|N)=(4-2)/(6-2)=2/4
  - p(italien|R)=.5, p(italien|N)=(4-1)/(6-2)=3/4
  - p(gart|R)=.5, p(gart|N)=(2-1)/(6-2)=1/4
  - p(miet|R)=.5, p(miet|N)=(1-1)/(6-2)=0~0.01

Smoothing: Avoid 1-1=0

Re-Ranking $rel(d,q) \approx \prod_{k \in d \cap q} \frac{p(k|R)*(1-p(k|N))}{p(k|N)*(1-p(k|R))}$

	V	H	I	G	M	B	W
1	1	1	1				
2		1		1	1		
3		1	1				
4			1	1			
5		1	1			1	
6	1					1	
Q		1	1	1	1		1

- New ranking
  - rel(1,q) = $\frac{\text{p(haus|R)*(1-p(haus|N))*p(italien|R)*(1-p(italien|N))}}{\text{p(haus|N)*(1-p(haus|R))*p(italien|N)*(1-p(italien|R))}}$
    = ...
  - rel(2,q) = ...
  - ...

Pros and Cons

- Advantages
  - Sound (probabilistic) framework
    - Many researchers feel more comfortable – explanations for all steps
    - But: Several steps are very heuristic
  - Results converge to most relevant docs (empirically shown)
    - Under the assumption that relevant docs are similar by sharing term distributions that are different from distributions in irrelevant docs
- Disadvantages
  - First guesses often are pretty bad – slow convergence
  - Assumes statistical independence of terms (as many methods)
  - “Has never worked convincingly better in practice” [MS07]

Probabilistic Model versus VSM with Rel. Feedback

- Published 1990 by Salton & Buckley
- Comparison based on various corpora
- Improvement after 1 feedback iteration

eingesetzte Methode		CACM 1033 Dok. 30 Anfr.	CISI 12684 Dok. 84 Anfr.	CRAN 1397 Dok. 225 Anfr.	INSPEC 1460 Dok. 112 Anfr.	MED 3204 Dok. 64 Anfr.	Durchschnitt
initiale Anfrage							
	Precision	0,1459	0,1184	0,1156	0,1368	0,3346	
IDE (dec hi)							
mit allen	Precision	0,2704	0,1742	0,3011	0,2140	0,6305	
Termen	Verbesserung	+86%	+47%	+160%	+56%	+88%	+87%
ausgewählte	Precision	0,2479	0,1924	0,2498	0,1976	0,6218	
Terme	Verbesserung	+70%	+63%	+116%	+44%	+86%	+76%
BIR-Modell							
mit allen	Precision	0,2289	0,1436	0,3108	0,1621	0,5972	
Termen	Verbesserung	+57%	+21%	+169%	+19%	+78%	+69%
ausgewählte	Precision	0,2224	0,1634	0,2120	0,1876	0,5643	
Terme	Verbesserung	+52%	+38%	+83%	+37%	+69%	+56%

- Probabilistic model (BIR) in general worse than VSM+rel feedback (IDE)
  - Probabilistic model does not weight terms in documents
  - Probabilistic model does not allow to weight terms in queries

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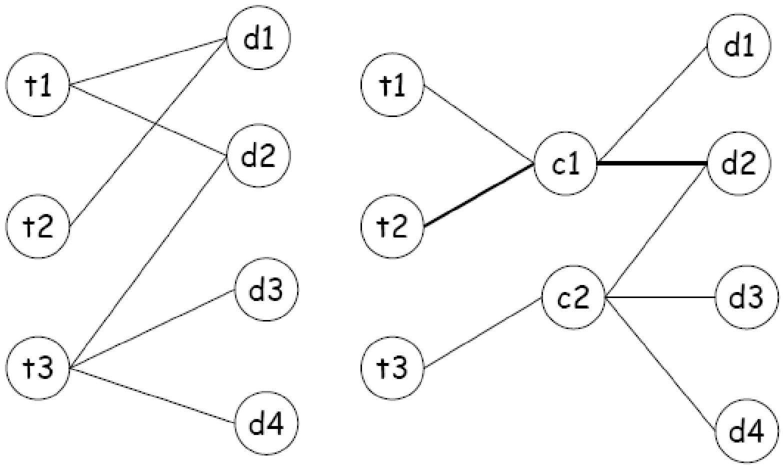
- Latent Semantic Indexing
- Other IR Models

Latent Semantic Indexing

- We so-far ignored semantic relationships between terms
  - Homonyms: bank (money, river)
  - Synonyms: House, building, hut, villa, ...
  - Hyperonyms: officer – lieutenant
- Idea of Latent Semantic Indexing (LSI)
  - Deerwester, S., Dumais, S. T., Furnas, G. W., Landauer, T. K. and Harshman, R. (1990). "Indexing by latent semantic analysis." Journal of the American society for information science 41(6): 391-407.
    - 2011: >7500 citations; 2014: ~9400
  - Map (many) terms into (fewer) semantic concepts
    - Which are hidden (or "latent") in the docs
  - Compare docs and query in concept space instead of term space
- May find docs that don't contain a single query term

Terms and Concepts



Quelle: K. Aberer, IR

- Concepts are more abstract than terms
- Concepts are related to terms and to docs
- LSI models concepts is non-exclusive sets of frequently co-occurring terms
  - Can be computing by matrix manipulations
  - Concepts from LSI cannot be "spelled out", but are matrix columns

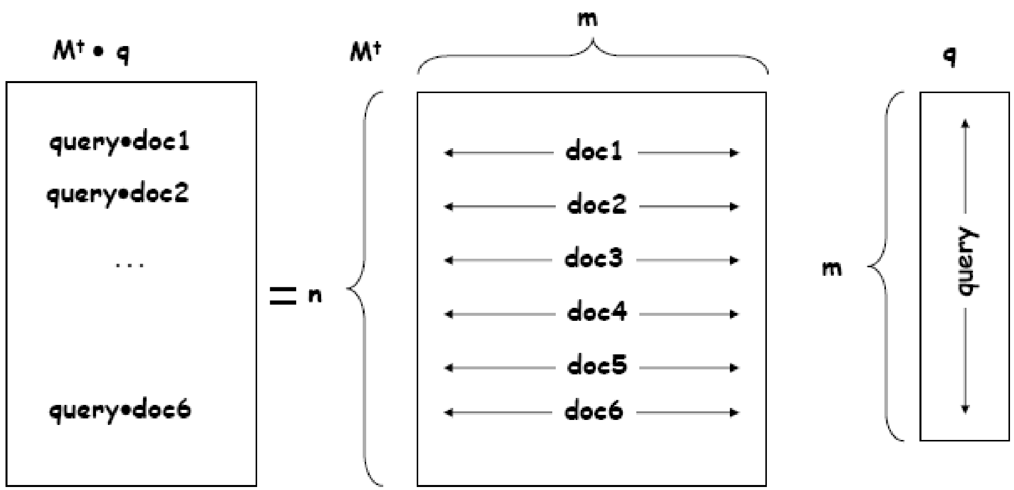
Term-Document Matrix

- Definition
  The term-document matrix M for docs D and terms K has n=|D| columns and m=|K| rows. M[i,j]=1 iff document d_j contains term k_i.
  - Works equally well for TF or TF*IDF values

Begriff	Dokument 1	Dokument 2	Dokument 3
Access	1	0	0
Document	1	0	0
Retrieval	1	0	1
Information	0	1	1
Theory	0	1	0
Database	1	0	0
Indexing	1	0	0
Computer	0	1	1

Term-Document Matrix and VSM

- VSM uses the transposed document-term matrix (=M^t)
- Having M, we can in principle compute the vector v containing the VSM-scores of all docs given q as $v=M^t \bullet q$
  - Computes the dot product, normalization missing



What to do with a Term-Document Matrix

- M is not just a comfortable way of representing the term vectors of all documents
- In the following, we approximate M by a particular M’
  - M’ should be smaller than M
    - Less dimensions; faster computations
  - M’ should abstract from terms to concepts
    - The fewer dimensions capture the most frequent co-occurrences
  - M’ should be such that $M'^t*q' \approx M^t*q$
    - Produce the least error among all M’ of the same dimension
- Note: We don’t delve deep into the math behind LSI

Term and Document Correlation

- $M \bullet M^t$ is called the term correlation matrix
  - Has |K| columns and |K| rows
  - "Similarity" of terms: how often do they co-occur in a doc?
- $M^t \bullet M$ is called the document correlation matrix
  - Has |D| columns and |D| rows
  - "Similarity" of docs: how many terms do they share?
- Example

	1	2	3	4	5
A	1	1	1		
B	1	1	1		1
C		1	1		
D				1	1

M

•

	A	B	C	D
1	1	1		
2	1	1	1	
3	1	1	1	
4				1
5		1		1

M^t

=

	A	B	C	D
A	3	3	2	0
B	3	4	2	1
C	2	2	2	0
D	0	1	0	2

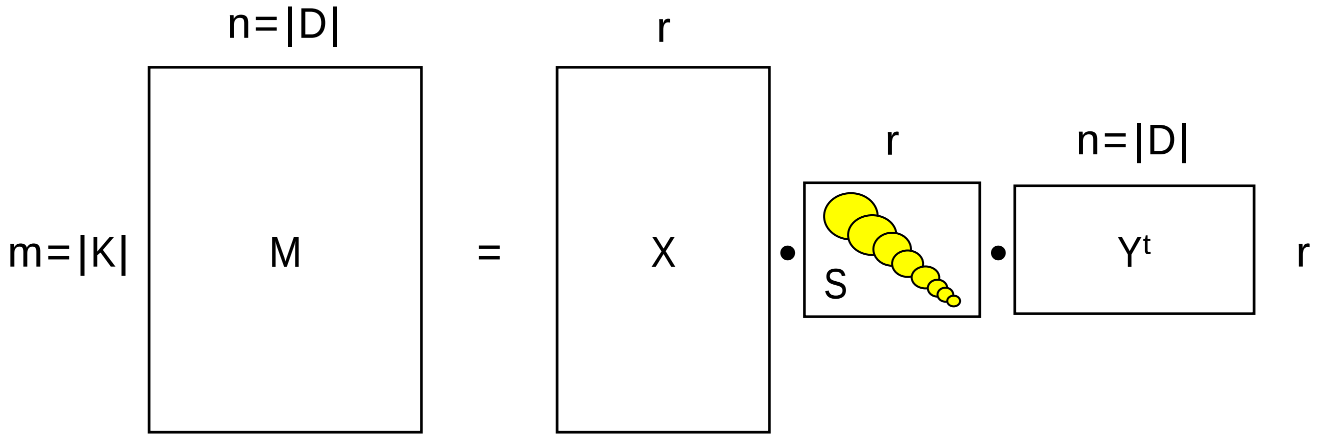
Term correlation matrix

Some Linear Algebra [Recap]

- The rank of a matrix M (r) is the maximal number of linearly independent rows of M
- If $Mx-\lambda x=0$ for a vector $x\neq 0$, then λ is called an Eigenvalue of M and x is his associated Eigenvector
  - Eigenvectors/-werte are useful for many things
  - In particular, a matrix M can be transformed into a diagonal matrix L with $L=U^{-1}*M*U$ with U formed from the Eigenvectors of M iff M has "enough" Eigenvectors
    - L represents M in another vector space, based on another basis
    - L can be used in many cases instead of M and is easier to handle
  - However, our M usually will not have "enough" Eigenvectors
  - We use another factorization of M

Singular Value Decomposition (SVD)

- SVD decomposes any matrix into $M = X \bullet S \bullet Y^t$
  – S is the diagonal matrix of the singular values of M in descending order and has size rxr (with r=rank(M))
  – X is the matrix of Eigenvectors of $M \bullet M^t$
  – Y is the matrix of Eigenvectors of $M^t \bullet M$
  – This decomposition is unique and can be computed in $O(r^3)$
    - Use approximations in practice



Example

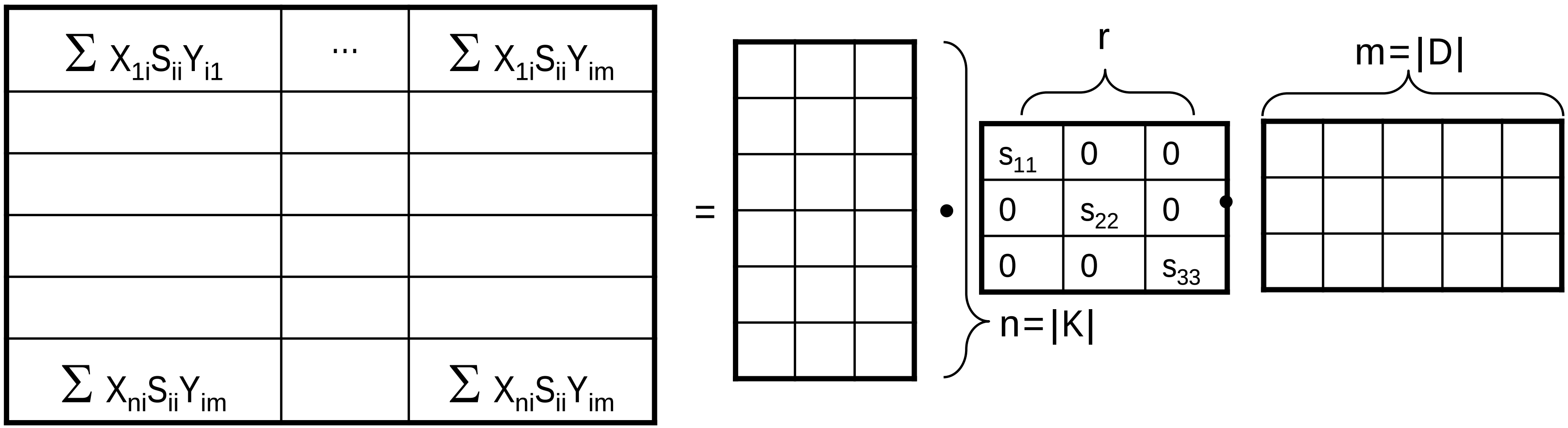
- Assume for now M is quadratic and has full rank
  - Example for r=|K|=|D|=3

$$
\begin{array}{|c|c|c|}\hline M_{11} & M_{12} & M_{13} \\ \hline M_{21} & \ldots & \ldots \\ \hline M_{31} & \ldots & M_{33} \\ \hline \end{array}
=
\begin{array}{|c|c|c|}\hline x_{11} & \ldots & \ldots \\ \hline x_{21} & \ldots & \ldots \\ \hline \ldots & \ldots & x_{33} \\ \hline \end{array}
\bullet
\begin{array}{|c|c|c|}\hline s_{11} & 0 & 0 \\ \hline 0 & s_{22} & 0 \\ \hline 0 & 0 & s_{33} \\ \hline \end{array}
\bullet
\begin{array}{|c|c|c|}\hline y_{11} & \ldots & \ldots \\ \hline y_{21} & \ldots & \ldots \\ \hline \ldots & \ldots & y_{33} \\ \hline \end{array}
$$

- $M_{11} = (x_{11}*s_{11}+x_{12}*s_{12}+x_{13}*s_{13})*y_{11} + (x_{11}*s_{21}+x_{12}*s_{22}+x_{13}*s_{23})*y_{21} + (x_{11}*s_{31}+x_{12}*s_{32}+x_{13}*s_{33})*y_{31}$
  $= x_{11}*s_{11}*y_{11} + x_{12}*s_{22}*y_{21} + x_{13}*s_{33}*y_{31}$
- $M_{12} = \ldots$

General Case

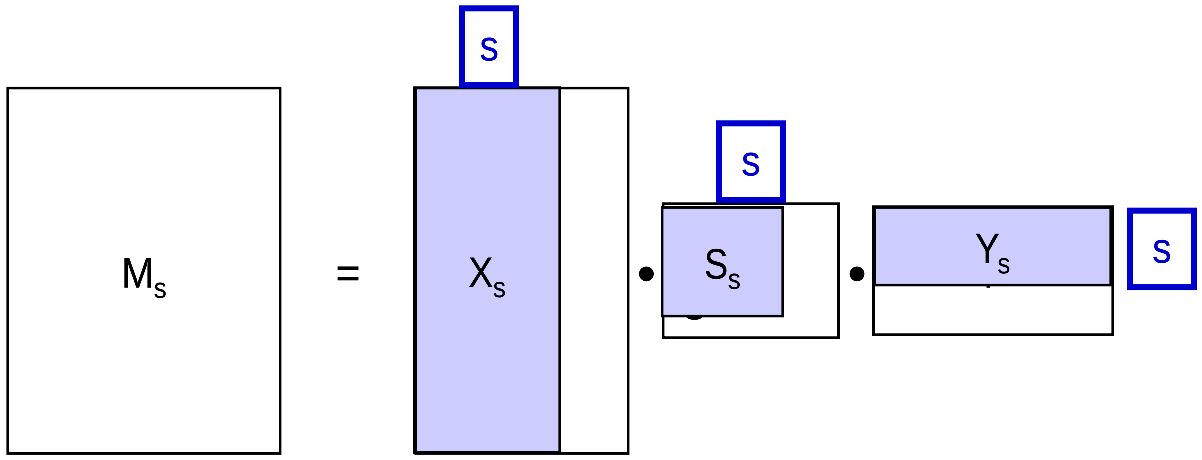
- In general, M is not quadratic; $r < \min(|K|,|D|)$
  - All sums range from 1 to r



- LSI idea: What if we stop the sums earlier, at some $s<r$?
  - s_{ii} are sorted by descending value
  - Aggregating only over the first s_{ii}–values captures "most" of M

Approximating M

- LSI: Use S to approximate M
- Fix some s<r; Compute $M_s = X_s \bullet S_s \bullet Y_s^t$
  - X_s : First s columns in X
  - S_s : First s columns and first s rows in S
  - Y_s : First s rows in Y
- M_s has the same size as M, but different values
  - In fact, we don't need to compute M_s, but only need X_s, S_s and Y_s



s-Approximations

- Formal: M_s is the matrix where $||M-M_s||_2$ is the smallest
- Since the s_{ii} are sorted in decreasing order
  - The approximation is the better, the larger s
  - The computation is the faster, the smaller s
- LSI: Only consider the top-s singular values
  - s must be small enough to filter out noise (spurious co-occurrences) and to provide "semantic reduction"
  - s must be large enough to represent the diversity in the documents
  - Typical value: 200-500

LSI for Information Retrieval

- We map document vectors from a m-dimensional space into a s-dimensional space
  - Approximated docs (still) are represented by columns in Y_s^t
- SVD as much as possible preserves distances between docs (depending on number of shared co-occurring terms)
- To this end, SVD (in a way) maps frequently co-occurring terms to the same dimensions
  - Because these terms have little impact on distance
- Frequently co-occurring terms can be seen as concepts
  - But they cannot easily be "named"
  - We cannot easily determine the terms that are mapped into a new dimension – it is always a bit of everything (a linear combination)

Query Evaluation

- After LSI, docs are represented by columns in Y_s^t
- How can we compute the distance between a query and a doc in concept space?
  - Transform q into concept space
  - Assume q as a new column in M
    - Of course, we can transform M offline, but need to transform q online
  - This would generate a new column in Y_s^t
  - To only compute this column, we apply the same transformations to q as we did to all other columns of M
  - With a little algebra, we get: $q' = q^t \bullet X_s \bullet S_s^{-1}$
  - This vector is compared to the transformed doc vectors as usual

Example: Term-Document Matrix

- Taken from Mi Islita: “Tutorials on SVD & LSI”
  - http://www.miislita.com/information-retrieval-tutorial/svd-lsi-tutorial-1-understanding.html
    - Who took if from the Grossman and Frieder book

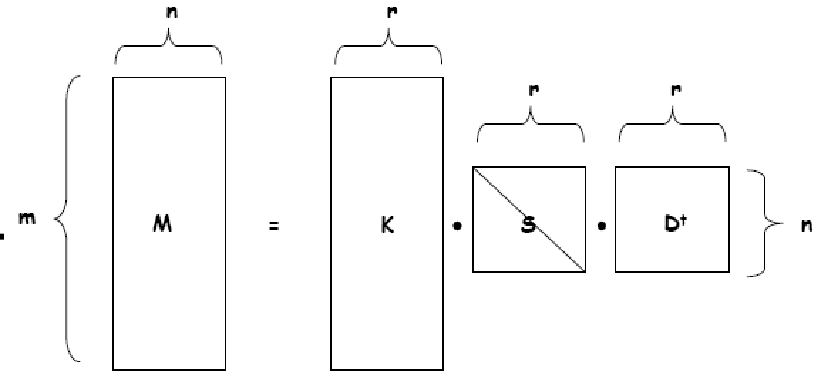
d1: *Shipment of gold damaged in a fire.*
d2: *Delivery of silver arrived in a silver truck.*
d3: *Shipment of gold arrived in a truck.*

Query: „gold silver truck“

Terms	d1	d2	d3	q
a	1	1	1	0
arrived	0	1	1	0
damaged	1	0	0	0
delivery	0	1	0	0
fire	1	0	0	0
gold	1	0	1	1
in	1	1	1	0
of	1	1	1	0
shipment	1	0	1	0
silver	0	2	0	1
truck	0	1	1	1

M = (columns d1, d2, d3), **q** = (column q)

Singular Value Decomposition



$$M = X \bullet S \bullet Y^t$$

$$X = \begin{bmatrix} -0.4201 & 0.0748 & -0.0460 \\ -0.2995 & -0.2001 & 0.4078 \\ -0.1206 & 0.2749 & -0.4538 \\ -0.1576 & -0.3046 & -0.2006 \\ -0.1206 & 0.2749 & -0.4538 \\ -0.2626 & 0.3794 & 0.1547 \\ -0.4201 & 0.0748 & -0.0460 \\ -0.4201 & 0.0748 & -0.0460 \\ -0.2626 & 0.3794 & 0.1547 \\ -0.3151 & -0.6093 & -0.4013 \\ -0.2995 & -0.2001 & 0.4078 \end{bmatrix} \quad S = \begin{bmatrix} 4.0989 & 0.0000 & 0.0000 \\ 0.0000 & 2.3616 & 0.0000 \\ 0.0000 & 0.0000 & 1.2737 \end{bmatrix}$$

$$Y = \begin{bmatrix} -0.4945 & 0.6492 & -0.5780 \\ -0.6458 & -0.7194 & -0.2556 \\ -0.5817 & 0.2469 & 0.7750 \end{bmatrix} \quad Y^t = \begin{bmatrix} -0.4945 & -0.6458 & -0.5817 \\ 0.6492 & -0.7194 & 0.2469 \\ -0.5780 & -0.2556 & 0.7750 \end{bmatrix}$$

A Two-Approximation (s=2)

$$X_2 = \begin{bmatrix} -0.4201 & 0.0748 \\ -0.2995 & -0.2001 \\ -0.1206 & 0.2749 \\ -0.1576 & -0.3046 \\ -0.1206 & 0.2749 \\ -0.2626 & 0.3794 \\ -0.4201 & 0.0748 \\ -0.4201 & 0.0748 \\ -0.2626 & 0.3794 \\ -0.3151 & -0.6093 \\ -0.2995 & -0.2001 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 4.0989 & 0.0000 \\ 0.0000 & 2.3616 \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} -0.4945 & 0.6492 \\ -0.6458 & -0.7194 \\ -0.5817 & 0.2469 \end{bmatrix} \qquad Y_2^t = \begin{array}{ccc} & & \\ \left[\begin{matrix} -0.4945 \\ 0.6492 \end{matrix}\right. & \begin{matrix} -0.6458 \\ -0.7194 \end{matrix} & \left.\begin{matrix} -0.5817 \\ 0.2469 \end{matrix}\right] \\ \uparrow & \uparrow & \uparrow \\ d_1 & d_2 & d_3 \end{array}$$

Transforming the Query

$$q' = q^t \bullet X_2 \bullet S_2^{-1}$$

$$q' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -0.4201 & 0.0748 \\ -0.2995 & -0.2001 \\ -0.1206 & 0.2749 \\ -0.1576 & -0.3046 \\ -0.1206 & 0.2749 \\ -0.2626 & 0.3794 \\ -0.4201 & 0.0748 \\ -0.4201 & 0.0748 \\ -0.2626 & 0.3794 \\ -0.3151 & -0.6093 \\ -0.2995 & -0.2001 \end{bmatrix} \begin{bmatrix} \frac{1}{4.0989} & 0.0000 \\ 0.0000 & \frac{1}{2.3616} \end{bmatrix}$$

$$= \begin{bmatrix} -0.2140 & -0.1821 \end{bmatrix}$$

Computing the Cosine of the Angle

$$\text{sim}(q, d) = \frac{q \bullet d}{|q|\,|d|}$$

$$\text{sim}(q, d_1) = \frac{(-0.2140)(-0.4945) + (-0.1821)(0.6492)}{\sqrt{(-0.2140)^2 + (-0.1821)^2}\,\sqrt{(-0.4945)^2 + (0.6492)^2}} = -0.0541$$

$$\text{sim}(q, d_2) = \frac{(-0.2140)(-0.6458) + (-0.1821)(-0.7194)}{\sqrt{(-0.2140)^2 + (-0.1821)^2}\,\sqrt{(-0.6458)^2 + (-0.7194)^2}} = 0.9910$$

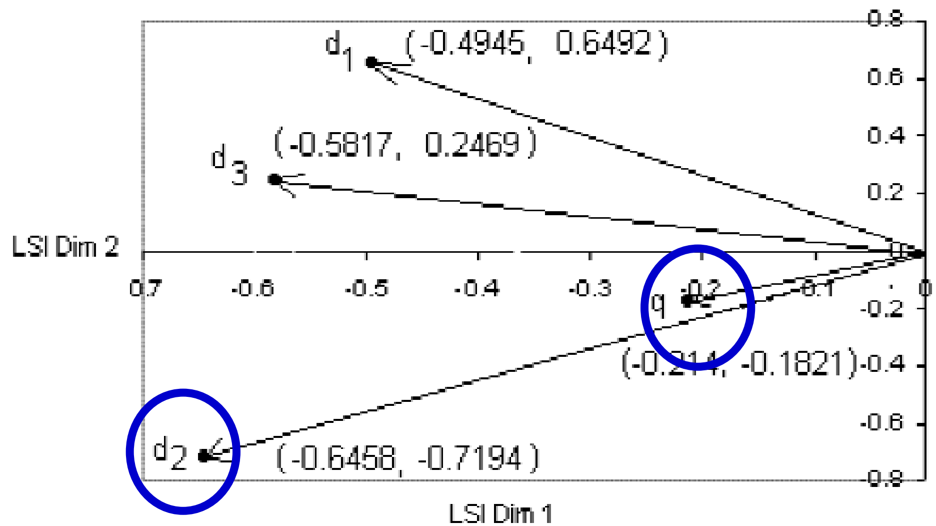
$$\text{sim}(q, d_3) = \frac{(-0.2140)(-0.5817) + (-0.1821)(0.2469)}{\sqrt{(-0.2140)^2 + (-0.1821)^2}\,\sqrt{(-0.5817)^2 + (0.2469)^2}} = 0.4478$$

Visualization of Results in 2D

M = , q =

Terms	d1	d2	d3	q
a	1	1	1	0
arrived	0	1	1	0
damaged	1	0	0	0
delivery	0	1	0	0
fire	1	0	0	0
gold	1	0	1	1
in	1	1	1	0
of	1	1	1	0
shipment	1	0	1	0
silver	0	2	0	1
truck	0	1	1	1

Very large distance in original space



Pros and Cons

- Pro
  - Made it into practice, but not if corpus is very large
    - [MPS08] claims: “no more than 1M docs”
  - May speed-up search due to less dimensions
  - Increases recall (and usually decreases precision)
- Contra
  - Computing SVD is expensive
    - Fast approximations exist, especially for extremely sparse matrices
    - Use stemming, stop-word removal etc. to shrink the original matrix
  - Ranking requires less dimensions than |D|, but more than |q|
    - Mapping the query turns a few keywords into an s-dimensional vector
    - We cannot simply index the “concepts” of M_s using inverted files etc.
    - Thus, LSI needs other techniques than inverted files
      - Means: lots of memory

Content of this Lecture

- IR Models
- Boolean Model
- Vector Space Model
- Relevance Feedback in the VSM
- Probabilistic Model
- Latent Semantic Indexing
- Other IR Models

Extended Boolean Model

- Critique to Boolean Model: If 1 conjunctive term out of 10 is missing, we get same result as if 10 were missing
- Idea: Measure "distance" for each conjunctive / disjunctive subterm of the query expression to the document
  - Example: X-ary AND: use a projection into x-dim space
  - Query expression is (1,1,1,...,1)
  - Doc is $(a_1,a_2,\ldots,a_x)=(0/1?,0/1?,\ldots)$
  - Similarity is distance between these two points
  - Other formulas for OR and NOT
- This model mimics the VSM
  - But no terms weights

Generalized Vector Space Model

- One critique to the VSM: Terms are not independent
- Thus, term vectors cannot be assumed to be orthogonal
- Generalized Vector Space Model
  - Build a much larger vector space with $2^{|K|}$ dimensions
    - Each dimension ("minterm") stands for all docs containing a particular set of terms
    - Minterms are not orthogonal but correlated by term co-occurrences
  - Convert query and docs into minterm space
  - Finally, rel(q, d) is the cosine of the angel in minterm space
- Nice theory, considers term co-occurrence, much more complex than ordinary VSM, no proven advantage

Self Assessment

- Explain the general approach of the probabilistic relevance model in IR
- How does one typically bootstrap this model?
- Which relevance model we discussed does consider the non-existent of terms in docs not existing in the query?
- Discuss the performance (speed) of the LSI approach to IR
- What is the difference between concept space and term space in LSI?
- Explain the Extended Boolean Model. Which of the shortcomings of the Boolean Model does it address?