Datenbanksysteme II:
Multidimensional Index Structures 1

Ulf Leser

Content of this Lecture

- Introduction
- Partitioned Hashing
- Grid Files
- kdb Trees
- R Trees

Multidimensional Indexing

- Access methods so far support access on attribute(s) for
  - Point query: Attribute = const (Hashing and B+ Tree)
  - Range query: $\text{const}_1 \leq \text{Attribute} \leq \text{const}_2$ (B+ Tree)
- What about more complex queries?
  - Point query on more than one attribute
    - Combined through AND (intersection) or OR (union)
  - Range query on more than one attribute
  - Queries for objects with size
    - “Sale” is a point in a multidimensional space
      - Time, location, product, ...
    - Geometric objects have size: rectangle, cubes, polygons, ...
  - Similarity queries: Most similar object, closest object, ...

Example: Geometric Objects

- Geographic information systems (GIS) store rectangles

```
RECT (X1, Y1, X2, Y2); x1<x2, y1<y2
```

- Typical GIS queries
  - Box query: All rectangles contained in query box (a1,b1)-(a2,b2)

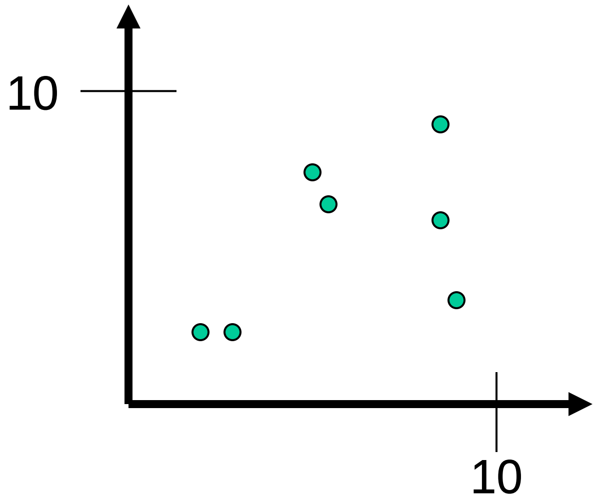
    ```
    SELECT * FROM RECT
    WHERE  a1 ≤ x1 and b1 ≤ y1 and
           a2 ≥ x2 and b2 ≥ y2
    ```

    - Results in a range query
  - Partial match query: Rectangles containing points with X=3

    ```
    SELECT * FROM RECT
    WHERE  X1 ≤ 3 and X2 ≥ 3
    ```

  - All rectangles with non-empty intersection with rectangle Q
- Also other shapes: Lines, polygons, 3D, ...

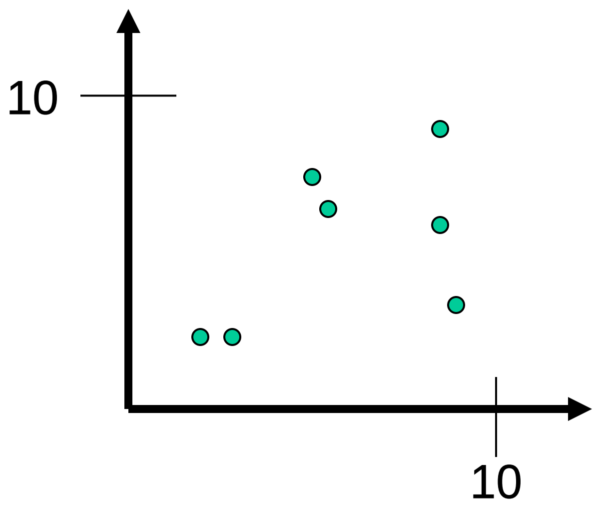
Example: 2D objects



Point	X	Y
P1	2	2
P2	2,5	2
P3	4,5	7
P4	4,7	6,5
P5	8	6
P6	8	9
P7	8,3	3

- Objects are points in a 2D space
- Queries
  - Exact: All objects with coordinates (X1, Y1)
  - Box: Find all points in a given rectangle
  - Partial: All points with X (Y) coordinate between ...

Option 1: Composite Index

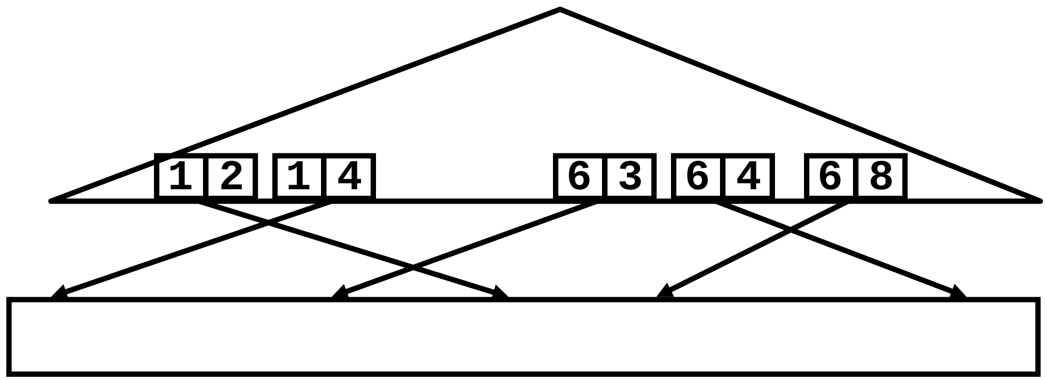



```
CREATE INDEX
 ON tab(x,y)
```

Point	X	Y
P1	2	2
P2	2,5	2
P3	4,5	7
P4	4,7	6,5
P5	8	6
P6	8	9
P7	8,3	3

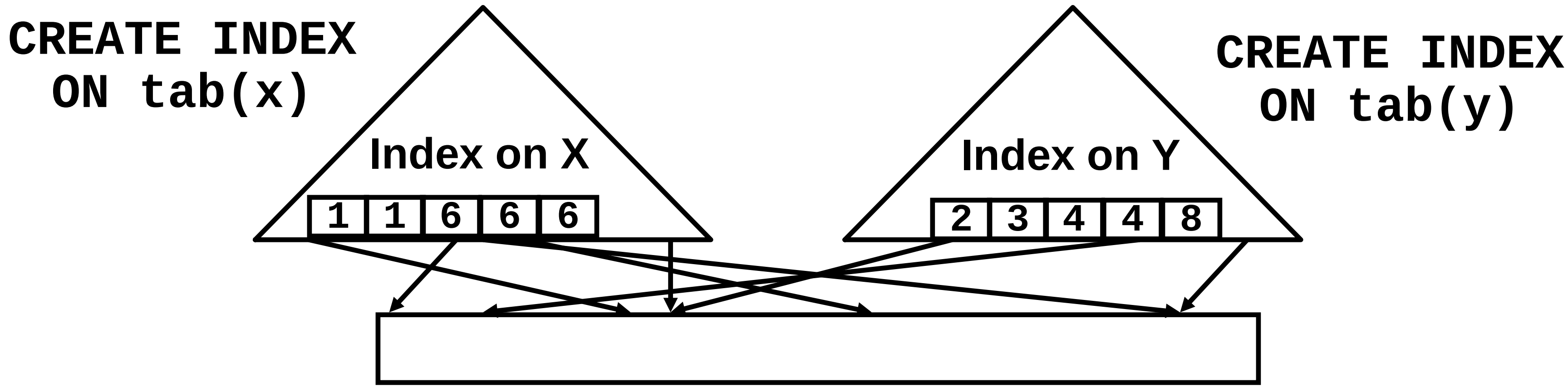
- Exact queries: Efficiently supported
- Box queries: Efficiently supported
- Partial match query
  - All points with X coordinate between ...: Efficiently supported
  - All points with Y coordinate between ...: Not efficiently supported

Composite Index



- Usage
  - Prefix of attribute list in index must be present in query
  - The longer the prefix, the more efficient the evaluation
- Alternatives
  - Also build index tab(Y, X) – all permutations
    - Combinatorial explosion for more than two attributes
  - Use independent indexes on each attribute

Option 2: Independent Indexes

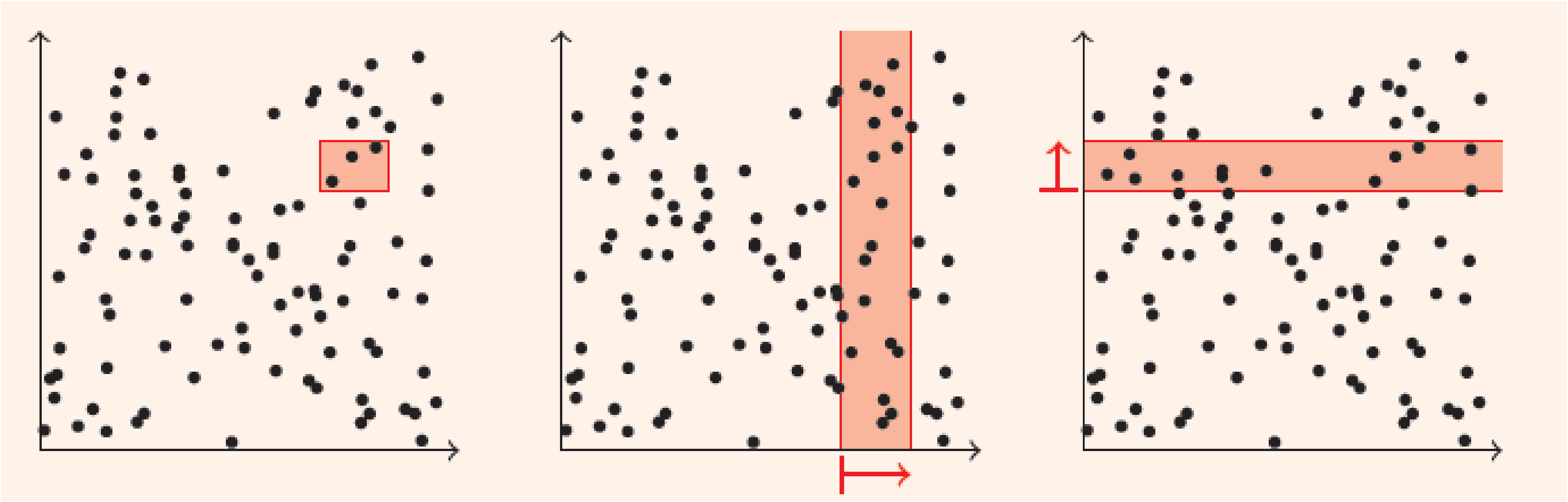


- Exact query: Not really efficient
  - Compute TID lists for each attribute
  - Intersect
- Box query: Not really efficient (compute ranges, intersect)
- Partial match query on one attribute: Efficiently supported

Example – Independent versus Composite Index

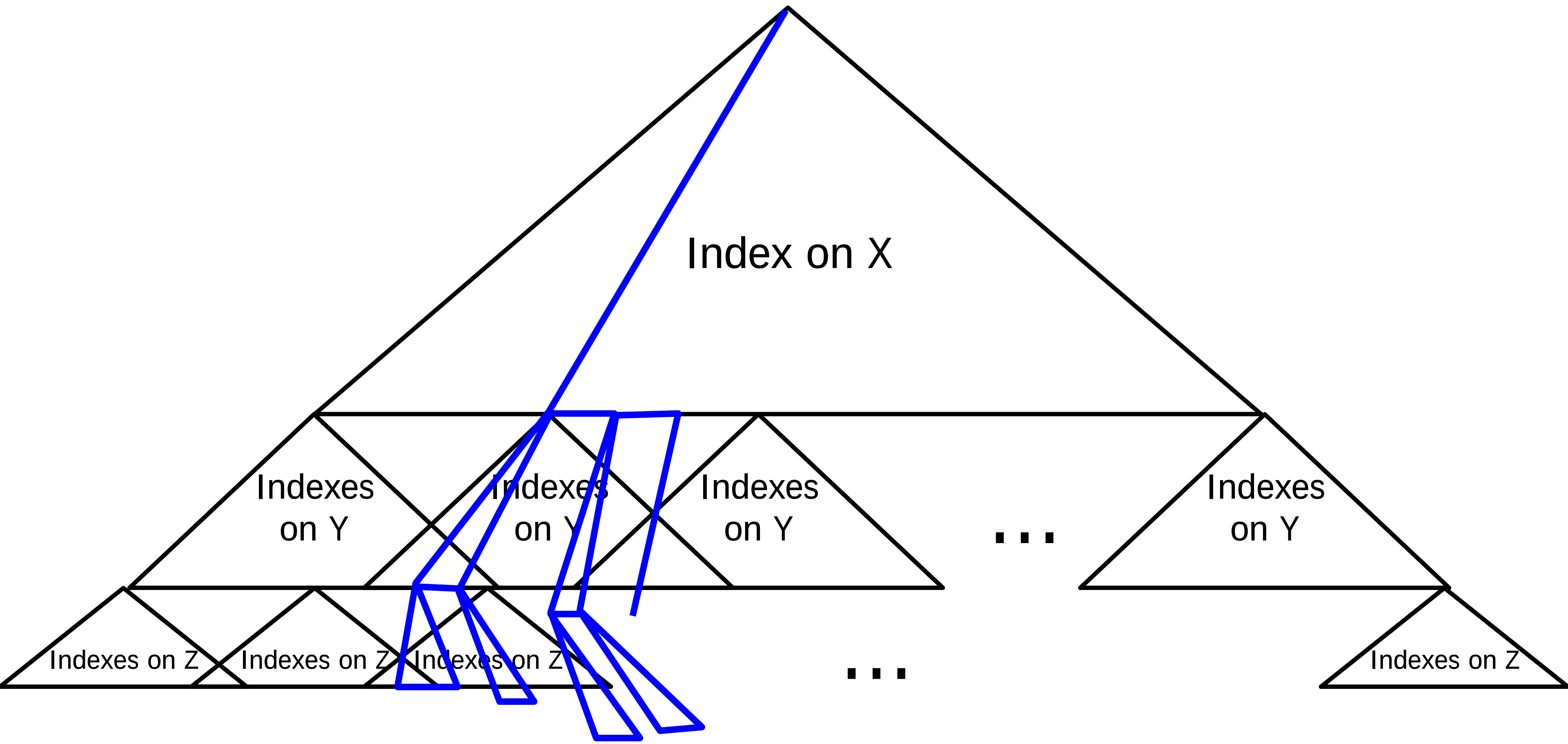
- Data
  - 3 dimensions of range 1,…,100
  - 1.000.000 points, randomly distributed
  - Index blocks holding 50 keys or records
- Assume three independent indexes
- Range query: Points with $40 \leq x \leq 50$, $40 \leq y \leq 50$, $40 \leq z \leq 50$
  - Each of the three B+-indexes has height 4
  - Using x-index, we generate TID-list |X|~100.000
  - Using y-index, we generate TID-list |Y|~100.000
  - Using z-index, we generate TID-list |Z|~100.000
  - For each index, we have 4+100.000/50=2004 IO
  - Hopefully, we can keep the three lists in main memory
  - Intersection yields app. 1.000 points, together 6012 IO

Intuition



Source: T. Grust, 2010

Composite Index



Using composite index (X,Y,Z)

- Key length increases – assume k=30 (or 10 / more dims)
- Index is higher: Height ~ 5 (6)
  - Worst case – index blocks only 50% filled
- We descend in 5 IO to leaves, read 10 points (1 IO), ascend to Y-axis (2 IO – but cached), descend to leaves (2 IO), read 10 points (1 IO) ...
- We do this 10*10 times
- Altogether
  - k=30 => app. 3+100*(2+1) ~ 303 IO
    - Compared to 6012 for independent indexes!
  - k=10 => app. 4+100*(3+1) ~ 404 IO
- But: More random IO

Conclusion

- We want composite indexes: Less IO
  - Benefit grows for highly selective queries
  - Bit: If selectivity is low, scanning of relation might be faster than any index (sequential versus random IO)
- For partial match queries, we would need to index all attribute combinations – not feasible
- Solution: Use multidimensional index structures (MDIS)
  - Should have no priority of pre-defined dimensions
  - Should adapt to different and changing data distribution
  - Essentially, we want nearby points being nearby on disk
    - In an ideal world, we would need only 1000/30~33 IO
  - Area of intensive research for decades

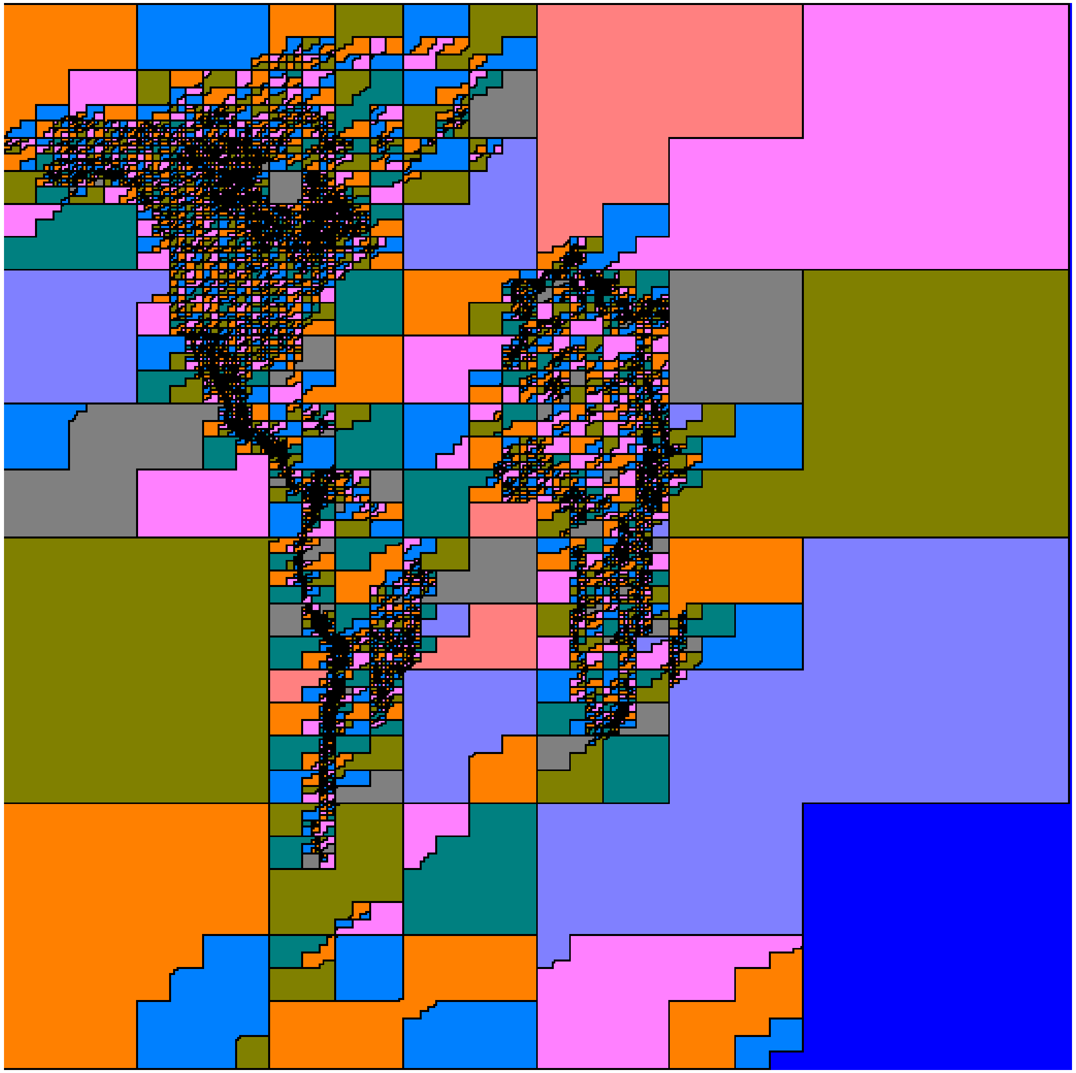
Multidimensional Indexes

- Specialized MDIS for objects with or without extend
- Critical issues
  - Balancing upon insert/delete: Worst case search complexity
  - Size: Amount of occupied space versus number of stored objects
  - Locality: Neighbors in space are stored nearby on disk (memory)
    - Necessary for range / partial match queries
    - Necessary for nearest neighbor queries
      - The nearest, all within distance k

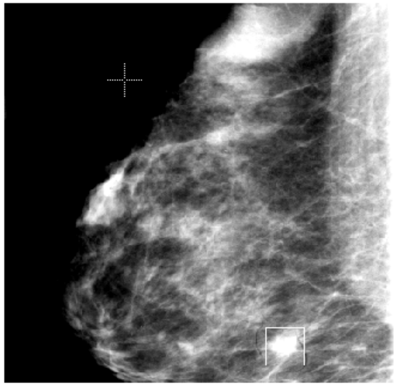
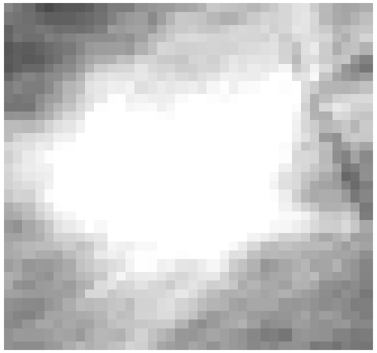
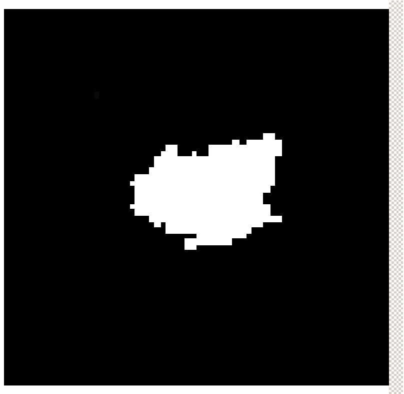
Caveats

- Things get complicated if data is not uniformly distributed
  - Dependent attributes (age – weight, income, height, ...)
  - Clustering of points
  - Also called skew – strong deviation from assumed distribution
- Curse of dimensionality: MDIS degrade for many dims
  - Trees difficult to balance, bad space usage, excessive management cost, expensive insertions/deletions, ...
- Alternative (partially): Bitmap indexes
  - Very small memory footprint, only for discrete attribute values, range queries become large disjunctions
- In commercial DBMS, high dim data is supported for
  - Geometric objects: GIS extensions, spatial extender
  - Multimedia data (images, songs, ...)

Geographic Information Systems



Multimedia Databases

- Map object into feature vector
  - Here: Tumor images; shapes derived from math. morphology
- Compute nearest neighborhood queries in feature space
  - Common approach: Filter away most objects as fast as possible
    - For instance by using shapes at different levels of granularity
  - Often, a final check of remaining candidates results is necessary

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Partitioned Hashing

- Let a_1 , a_2 ,…, a_d be the attributes to be indexed
- Define a hash function h_i for each a_i generating a bitstring
- Definition
  - Let $h_i(a_i)$ map each a_i into a bitstring of length b_i
  - Let $b=\sum b_i$ (length of global hash key in bits)
  - The global hash function $h(v_1 , v_2 , . . . , v_d) \rightarrow [0, …, 2^b-1]$

    is defined as $h(v_1 , v_2 , . . . , v_d) = h_1(v_1) \oplus h_2(v_2) \oplus ... \oplus h_k (v_d)$
- We need $B = 2^b$ buckets
  - Static address space – dynamic structures later

Example

- Data: (3,6),(6,7),(1,1),(3,1),(5,6),(4,3),(5,0),(6,1),(0,4),(7,2)
- Let h_1, h_2 be (b_1=b_2=1)

$$h_i\ (v_i) = \begin{cases} 0 & \text{if } 0 \leq v_i \leq 3 \\ 1 & \text{otherwise} \end{cases}$$

- Four buckets with addresses 00, 01, 10, 11

a_2

	0	1
0	**(1,1) (3,1)**	**(3,6) (0,4)**
1	**(4,3) (5,0) (6,1) (7,2)**	**(6,7) (5,6)**

a_1

Queries with Partitioned Hashing

- Exact point queries: Direct access to bucket
  - All points in bucket are candidates; check identity to query
- Partial match queries
  - Only parts of the global hash key are determined
  - Use those as filter; scan all buckets passing the filter
  - Let c be the number of unspecified bits
    - Then 2^c buckets must be searched
    - These are certainly not ordered on disk– random IO
- Range queries
  - Not efficiently supported, if hash function doesn't preserve order
    - Not order preserving: modulo; order preserving: division
  - Enumerate all in-between values, blocks will be anywhere

Order Preserving Hash Function

- Example
  - Suppose d=3, each dim with range 1..1024 (10 bits)
  - Use three highest bits as hash keys in each dimension
    - Order preserving; equal to division by 64 (right-shift 7 times)
  - Global hash key: 9 bit, hence 2^9=512 buckets
  - Partial range query: points with $200<y<300$ and $z<600$
    - $h_y(200)$=0011001000, $h_y(300)$=0100101100, $h_z(600)$=1001011000
    - Scan buckets with
      - X-coordinate: ?
      - Y-coordinate: between 001 and 010 (001, 010)
      - Z-coordinate: less than 100... (000, 001, 010, 011,100)
    - We need to scan 8 (x) * 2 (y) * 5(z) = 80 buckets
- Vulnerable to not-uniformly distributed data
  - Few buckets are extremely full, others empty

Without oph:
Enumerate all
values in DB and
compute hashkeys

Partitioned Hashing: Conclusions

- No balancing, no adaptation to skew
  - Long overflow buckets or large directories (see ext/lin hashing)
- Size: Static size of index, no adaptation
  - Problem if buckets overflow
  - Can be combined with extensible/linear hashing
  - Directory in extensible hashing can grow quite large
- Locality: Neighboring points in space not nearby in index
  - Usually, hash functions are not order preserving to achieve more uniform spread
  - Bad support for (partial) range queries or nearest neighbor queries

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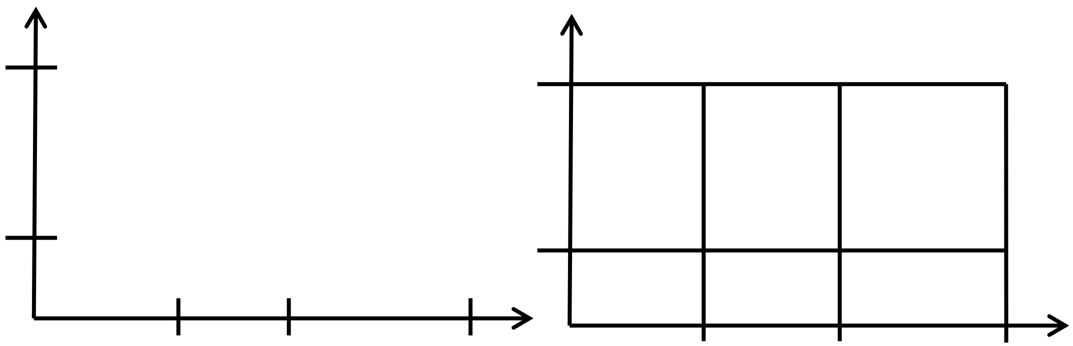
Grid File

- Classical multidimensional index structure
  - Nievergelt, J., Hinterberger, H. and Sevcik, K. C. (1984). "The Grid File: An Adaptable, Symmetric Multikey File Structure." *ACM TODS*
  - Conceptually simple
  - Can be seen as extensible version of partitioned hashing
  - Good for uniformly distributed data, bad for skewed data
  - Numerous variations, we only look at the basic method
- Design goals
  - Support exact, partial match, and neighbor queries
  - Guarantee “two IO” access to each point
    - Under certain assumptions
  - Adapt dynamically to the number of points

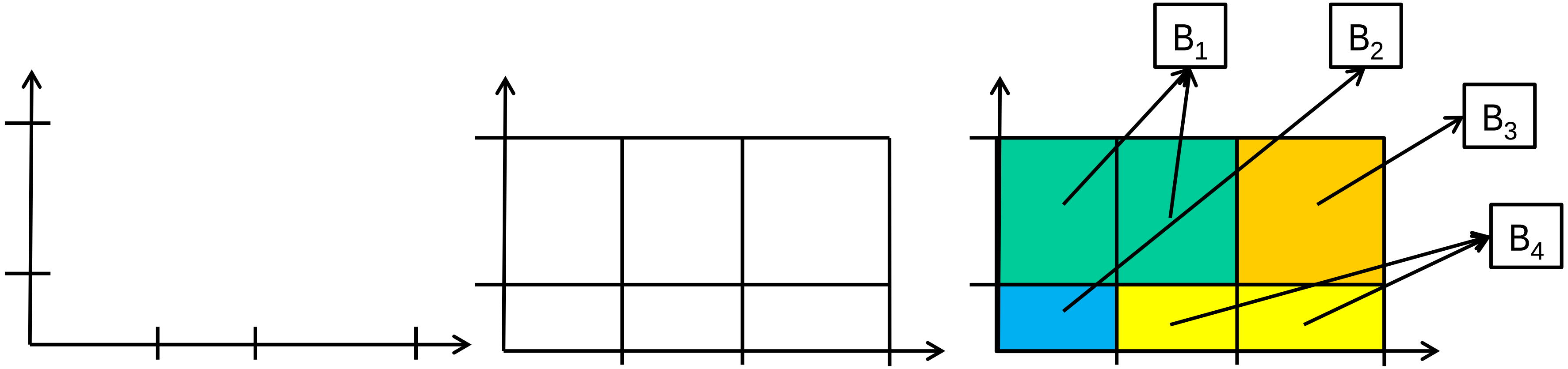
Principle

- Partition each dimension into disjoint intervals (scales)
  - EXCESS: Uniform scales; less adaptive, no scale management
- Intersection of all intervals defines grid cells
  - d-dimensional hypercubes
- Grid cells are addressed from the grid directory (GD)



Principle

- Partition each dimension into disjoint intervals (scales)
- Intersection of all intervals defines grid cells
- Grid cells are addressed from the grid directory (GD)
- Cells are grouped in regions; region = bucket = block
  - When multi-cell region overflows – split into cells
  - When single-cell region overflows – new scale, change GD
- Buckets hold coordinates + TID



Exact Point Search

- Assumption: GD in main memory
  - Size: $|S_1|*|S_2|*...|S_d|$, when S_i is the set of scales for dimension I
  - Cannot work for really high dimensional data

- 1. Compute grid cell
  - Look-up coordinates in scales to obtain GD coordinates
  - Cell in GD contains bucket address on disk
  - Bucket contains all data points in this grid cell (maybe more)
- 2. Load bucket and find point(s): 1^{st} IO
  - As usual, we do not look at how to search inside bucket
- 3. Access record following TID: 2^{nd} IO

Other Queries

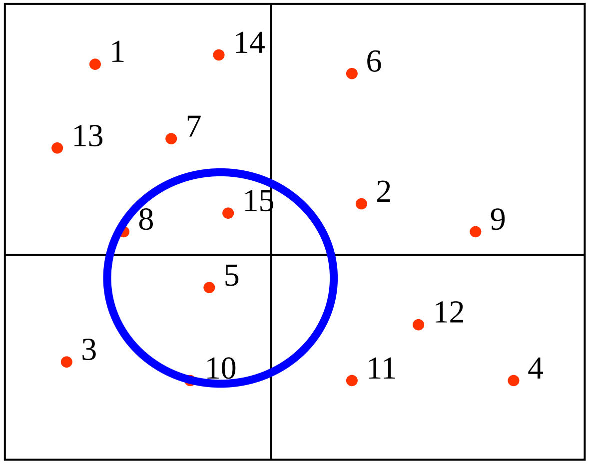
- Range query
  - Compute all matching scales
  - Access all corresponding cells in GD
  - Load and search all buckets (random IO)
- Partial match query
  - Compute partial GD coordinates
  - All GD cells with these coordinates may contain points (random IO)

Nearest Neighbor Queries

- Find bucket containing query point
- Search points in this region and choose closest
  - Can we finish if closest point was found?

Nearest Neighbor Queries

- Find bucket containing query point
- Check points in this region
  - Can we finish with the closest point in this region?
  - Usually not
    - Check distances to all borders
    - If point found is closer than any border, we are done
    - Otherwise, we need to search neighboring regions
    - Do it iteratively and always adapt radius to current closest point
  - Very fast if neighbor is in same region
    - I.e.: dense buckets and point not at border

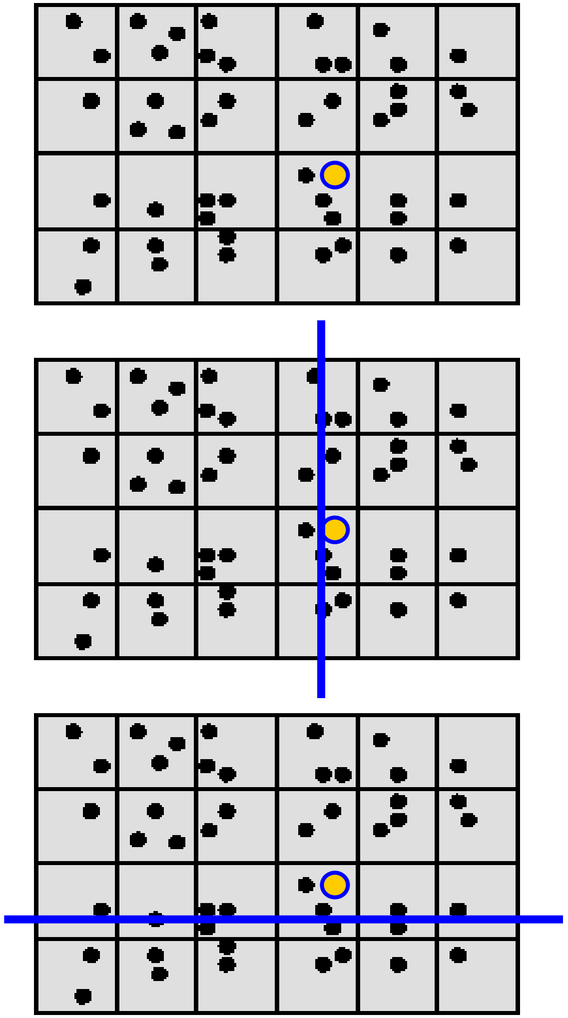


Inserting Points

- Search grid cell; if bucket has space: Insert point
- Otherwise (overflow): Split
  - Assume we have to split a single-cell region
  - Choose a dimension and new scale within region interval
  - Split all affected GD cells – cuts through all dimensions
    - Consider n dimensions and S_i scales in dimension i
    - Split in dim i affects $d_1 * \ldots * d_{i-1} * d_{i+1} * \ldots * d_n$ cells in GD
    - Example: d=3, S_i=4; $|GD|=4^3=64$; any split affects 4^2 cells
  - Split overflown bucket along new scale (new region)
  - Do not split other (un-overflown) buckets containing the new scale
    - Only copy pointer from GD to bucket
  - Choice of dimension and interval is difficult
    - Optimally, we would like to split as many rather full blocks as possible
    - We also want to consider our future expectation

Example

- Imagine one block holds 3 points
  - [Usually scales are unevenly spaced]
- New point causes overflow
- Vertical split
  - Splits 2 (3,4)-point blocks
  - Leaves one 3-point block
- Horizontal split
  - Splits 2 (3,4)-point blocks
  - Leaves one 3-point block
- Note: Most splits will happen only in the future
  - Creating more or less problems

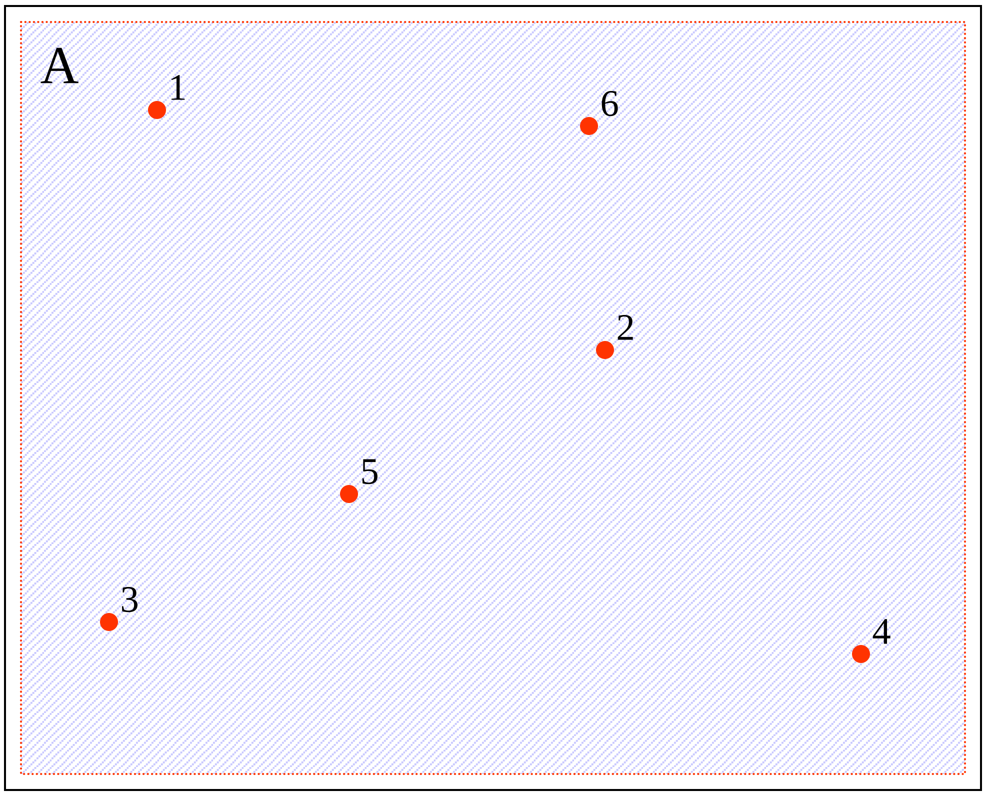


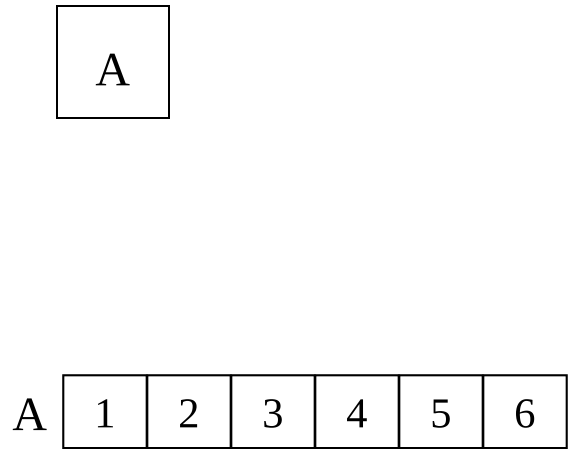
Inserting Points in Multi-Cell Regions

- Overflow in a multi-cell region
- Idea: Split region into smaller regions (or cells)
  - Possible split dimensions/axes: Existing scales not yet used for split in this region
    - No local adaptation – decisions from the past have to be obeyed
- Several strategies
  - Chose scale which best distributes the points
    - Requires considering them all
    - Won't pay off in case of uniformly distributed data
  - Circulate through dimensions and chose median scale

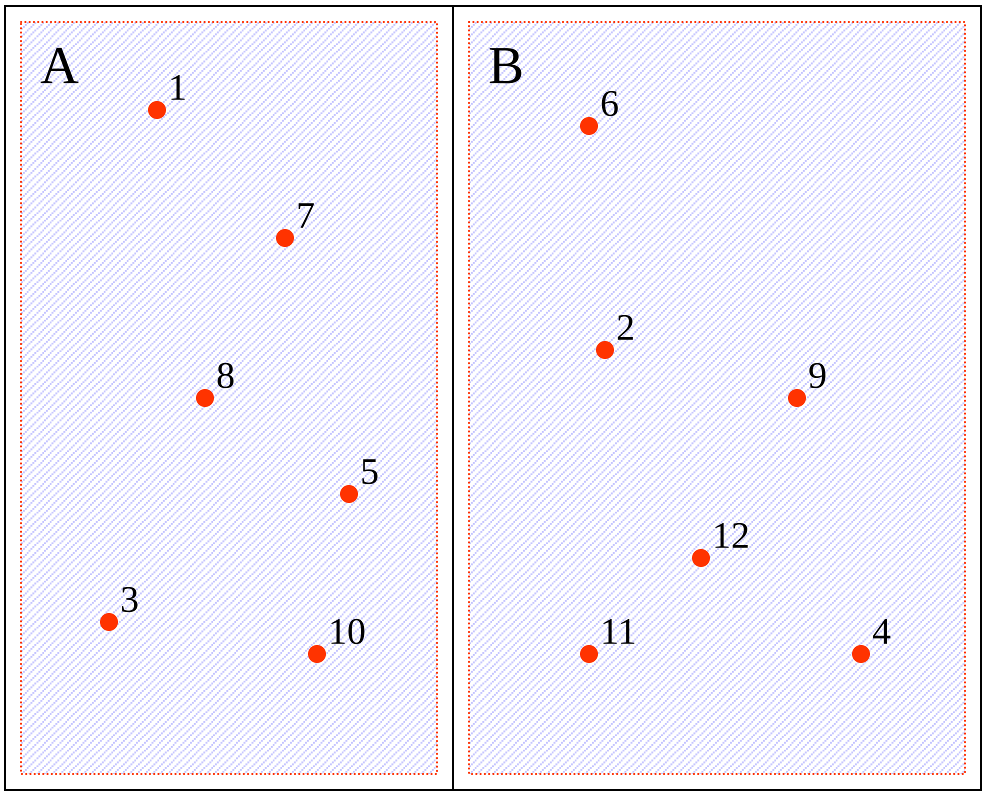
Grid File Example 1 [J. Gehrke]

Assume k=6





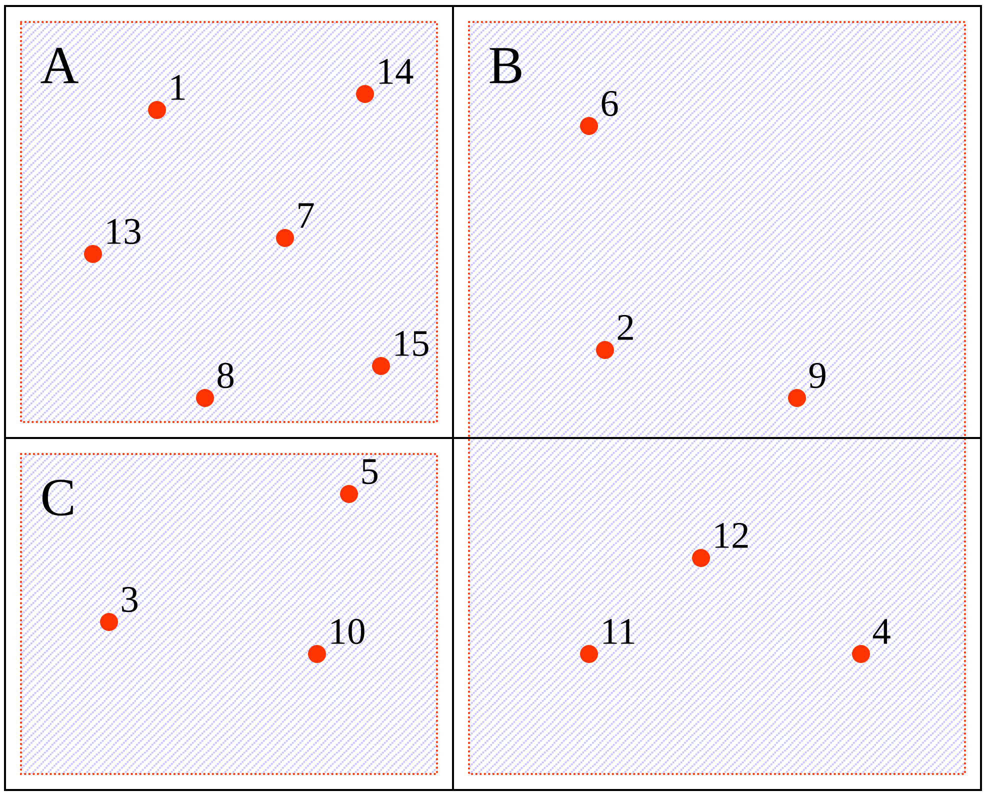
Grid File Example 2



A	B

A	1	3	5	7	8	10
B	2	4	6	9	11	12

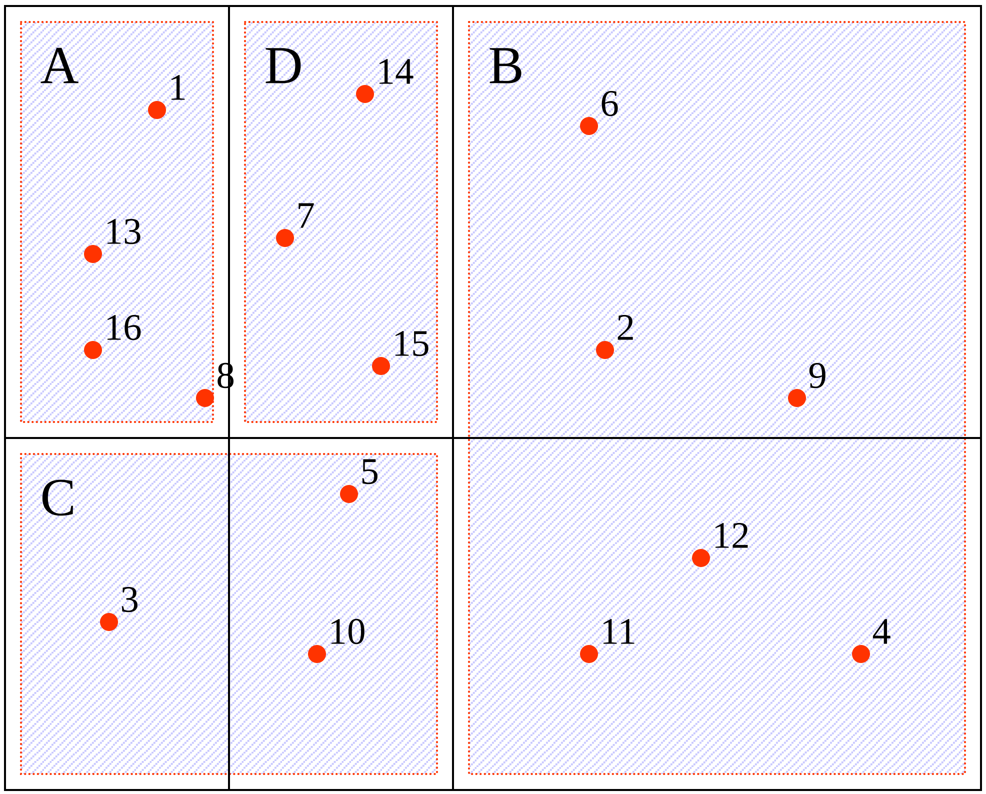
Grid File Example 3



A	B
C	B

A	1	7	8	13	14	15
B	2	4	6	9	11	12
C	3	5	10			

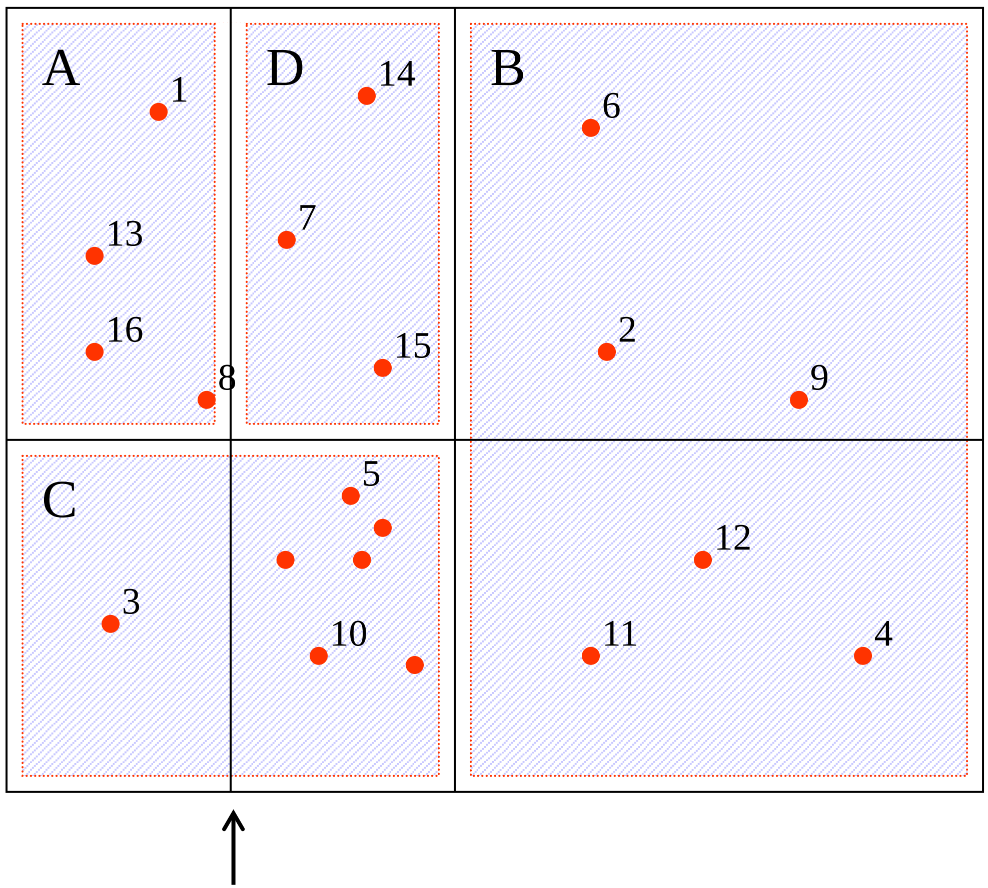
Grid File Example 4



A	D	B
C	C	B

A	1	8	13	16		
B	2	4	6	9	11	12
C	3	5	10			
D	7	14	15			

One Future

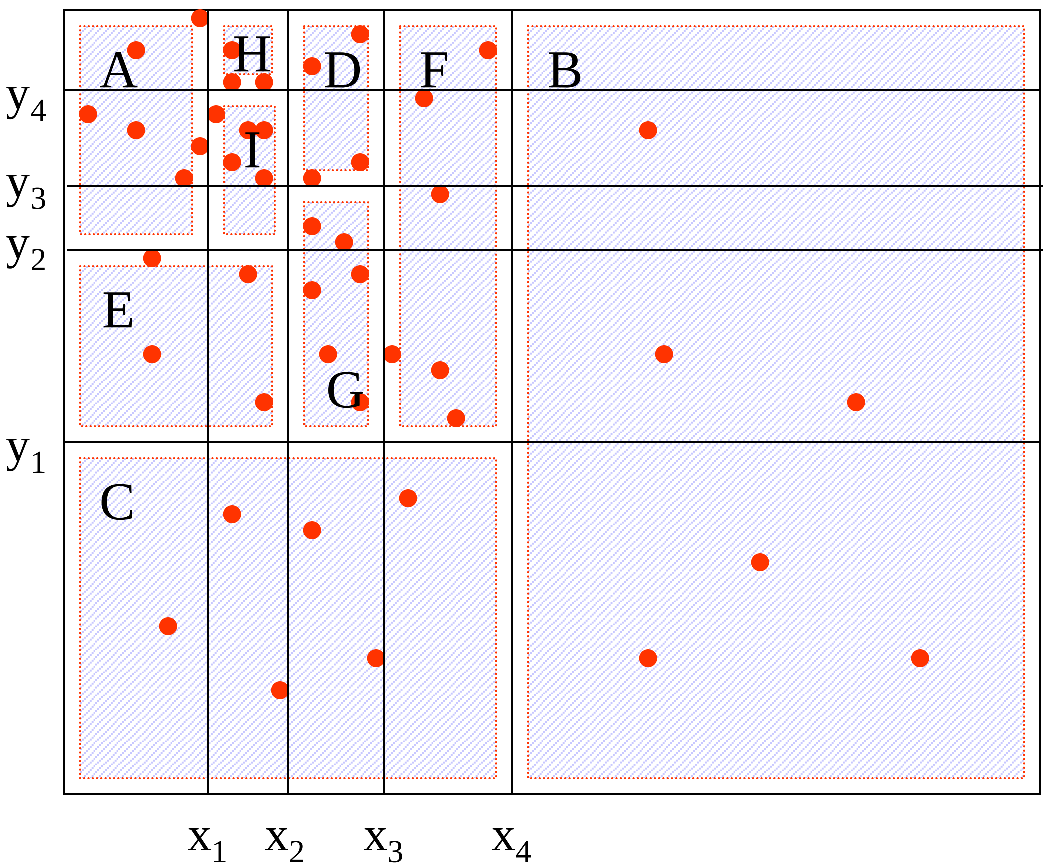


A	D	B
C	C	B

A	1	8	13	16		
B	2	4	6	9	11	12
C	3	5	10			
D	7	14	15			

We now must perform this split; creates one almost empty and one full bucket; next split will happen soon

Grid File Example 5



A	H	D	F	B
A	I	D	F	B
A	I	G	F	B
E	E	G	F	B
C	C	C	C	B

Deleting Points

- Search point and delete
- If block become ”almost empty”, try to merge
  - A merge is the removal of a split – chose scale to “unmake”
  - Should build larger convex regions
  - This can become very difficult
    - Potentially, more than two regions need to be merged to keep convexity
  - Eventually, also scales may be removed
    - Shrinkage of GD
  - Example: Where can we merge?

A	H	D	F	B
A	I	D	F	B
A	I	G	F	B
E	E	G	F	B
C	C	C	C	B

Convex Regions

A	H	D	F	B
A	I	D	F	B
A	I	G	F	B
E	E	G	F	B
C	C	C	C	B

A	I	D	F	B
A	I	D	F	B
A	I	G	F	B
E	E	G	F	B
C	C	C	C	B

- Non-convex regions: Range and neighborhood queries have to scan increasingly many buckets

A	A	D	F	B
A	I	D	F	B
A	I	G	F	B
E	E	G	F	B
C	C	C	C	B

What‘s in a Bucket?

- The tuples
  - Not compatible with other database structures (indexes, etc.)
  - Long tuples result in few records per data blocks
  - Frequent splits, fast growing GD
- Only TIDs
  - Many records per data block, few splits, small directory
  - But queries need to check (load) all tuples referenced in a block to check real coordinates
- TIDs and coordinates
  - Medium number of records per block, moderate growth of GD
  - No access to tuples necessary for checking coordinates

Original 2-IO Guarantee

- Assume GD on disk and buckets containing entire tuples
  – One more IO for loading pointer to bucket
  – One less IO for accessing payload
- But
  – Modern machines have large memories: GD in memory
    - For many dimensions, Grid files are anyway the wrong MDIS
  – More TIDs per bucket creates overall smaller data structure
  – Payload management (growing values etc.) independent of MDIS

Some Observations

- Grid files always split at hyperplanes parallel to the dimension axes
  - This is not always optimal
  - Use other bounding shapes: circles, polygons, etc.
  - More complex– forms might not disjointly fill the space any more
  - Allow overlaps (see R trees)
- There is no guaranteed block-fill degree – degeneration
- Choosing a new scale is a local decision with global consequences
  - No local adaptation: GD grows very fast
  - Need not be realized immediately, but restricts later choices in other regions
  - Bad adaptation to skewed data