

PAM and BLAST

Ulf Leser

This Lecture

- Substitution Matrices
 - PAM distance
 - PAM matrices
- Scaling up Local Alignments
 - BLAST

Substitution Matrices

- Recall

- A **scoring function** is a function $s: \Sigma \times \Sigma \rightarrow \text{Integer}$

- We also call s a substitution matrix

- Direct similarity

$$\text{sim}'(A, B) = \sum_{i=1}^n s(A[i], B[i])$$

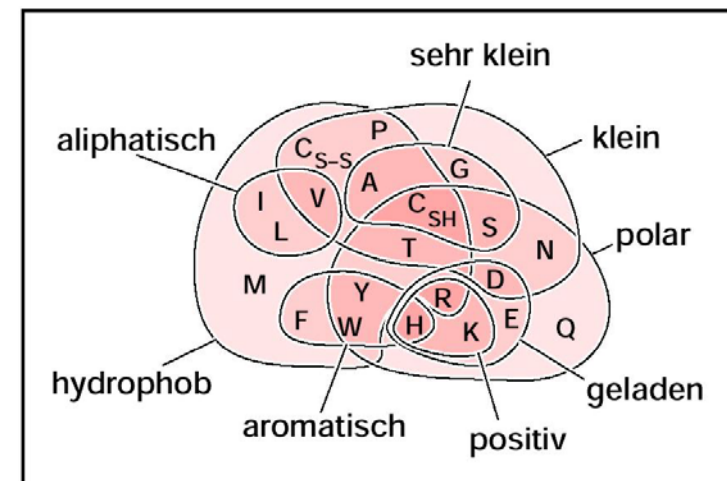
- DNA: symmetric, simple matrices

- **Protein sequences** are different

- Very heterogeneous properties

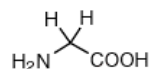
- Very different impact on folding

- Substitutions **may change the 3D structure completely** or not at all

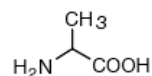


Amino Acids

Small

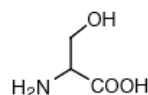


Glycine (Gly, G)
MW: 57.05

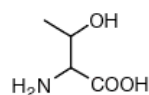


Alanine (Ala, A)
MW: 71.09

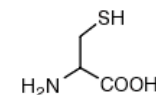
Nucleophilic



Serine (Ser, S)
MW: 87.08, $pK_a \sim 16$

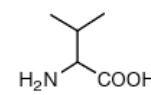


Threonine (Thr, T)
MW: 101.11, $pK_a \sim 16$

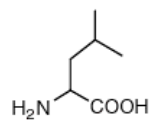


Cysteine (Cys, C)
MW: 103.15, $pK_a = 8.35$

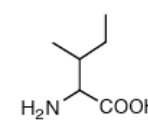
Hydrophobic



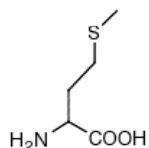
Valine (Val, V)
MW: 99.14



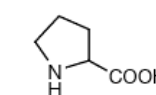
Leucine (Leu, L)
MW: 113.16



Isoleucine (Ile, I)
MW: 113.16

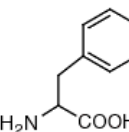


Methionine (Met, M)
MW: 131.19

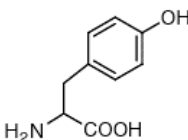


Proline (Pro, P)
MW: 97.12

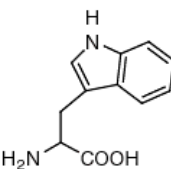
Aromatic



Phenylalanine (Phe, F)
MW: 147.18

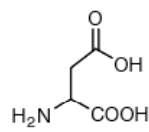


Tyrosine (Tyr, Y)
MW: 163.18

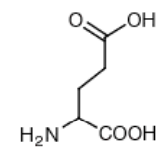


Tryptophan (Trp, W)
MW: 186.21

Acidic

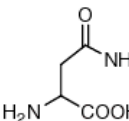


Aspartic Acid (Asp, D)
MW: 115.09, $pK_a = 3.9$

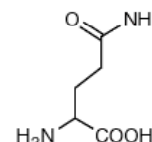


Glutamic Acid (Glu, E)
MW: 129.12, $pK_a = 4.07$

Amide

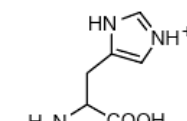


Asparagine (Asn, N)
MW: 114.11

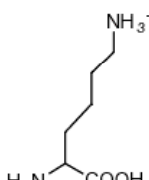


Glutamine (Gln, Q)
MW: 128.14

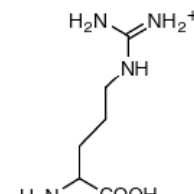
Basic



Histidine (His, H)
MW: 137.14, $pK_a = 6.04$



Lysine (Lys, K)
MW: 128.17, $pK_a = 10.79$



Arginine (Arg, R)
MW: 156.19, $pK_a = 12.48$

Example

Where
do
all
these
numbers
come
from?

	A	R	N	D	C	Q	E	G	H	I	L	K	M	F	P	S	T	W	Y	V	B	Z
A	4	-1	-2	-2	0	-1	-1	0	-2	-1	-1	-1	-1	-2	-1	1	0	-3	-2	0	-2	-1
R	-1	5	0	-2	-3	1	0	-2	0	-3	-2	2	-1	-3	-2	-1	-1	-3	-2	-3	-1	0
N	-2	0	6	1	-3	0	0	0	1	-3	-3	0	-2	-3	-2	1	0	-4	-2	-3	3	0
D	-2	-2	1	6	-3	0	2	-1	-1	-3	-4	-1	-3	-3	-1	0	-1	-4	-3	-3	4	1
C	0	-3	-3	-3	9	-3	-4	-3	-3	-1	-1	-3	-1	-2	-3	-1	-1	-2	-2	-1	-3	-3
Q	-1	1	0	0	-3	5	2	-2	0	-3	-2	1	0	-3	-1	0	-1	-2	-1	-2	0	3
E	-1	0	0	2	-4	2	5	-2	0	-3	-3	1	-2	-3	-1	0	-1	-3	-2	-2	1	4
G	0	-2	0	-1	-3	-2	-2	6	-2	-4	-4	-2	-3	-3	-2	0	-2	-2	-3	-3	-1	-2
H	-2	0	1	-1	-3	0	0	-2	8	-3	-3	-1	-2	-1	-2	-1	-2	-2	2	-3	0	0
I	-1	-3	-3	-3	-1	-3	-3	-4	-3	4	2	-3	1	0	-3	-2	-1	-3	-1	3	-3	-3
L	-1	-2	-3	-4	-1	-2	-3	-4	-3	2	4	-2	2	0	-3	-2	-1	-2	-1	1	-4	-3
K	-1	2	0	-1	-3	1	1	-2	-1	-3	-2	5	-1	-3	-1	0	-1	-3	-2	-2	0	1
M	-1	-1	-2	-3	-1	0	-2	-3	-2	1	2	-1	5	0	-2	-1	-1	-1	-1	1	-3	-1
F	-2	-3	-3	-3	-2	-3	-3	-3	-1	0	0	-3	0	6	-4	-2	-2	1	3	-1	-3	-3
P	-1	-2	-2	-1	-3	-1	-1	-2	-2	-3	-3	-1	-2	-4	7	-1	-1	-4	-3	-2	-2	-1
S	1	-1	1	0	-1	0	0	0	-1	-2	-2	0	-1	-2	-1	4	1	-3	-2	-2	0	0
T	0	-1	0	-1	-1	-1	-1	-2	-2	-1	-1	-1	-1	-2	-1	1	5	-2	-2	0	-1	-1
W	-3	-3	-4	-4	-2	-2	-3	-2	-2	-3	-2	-3	-1	1	-4	-3	-2	11	2	-3	-4	-3
Y	-2	-2	-2	-3	-2	-1	-2	-3	2	-1	-1	-2	-1	3	-3	-2	-2	2	7	-1	-3	-2
V	0	-3	-3	-3	-1	-2	-2	-3	-3	3	1	-2	1	-1	-2	-2	0	-3	-1	4	-3	-2
B	-2	-1	3	4	-3	0	1	-1	0	-3	-4	0	-3	-3	-2	0	-1	-4	-3	-3	4	1
Z	-1	0	0	1	-3	3	4	-2	0	-3	-3	1	-1	-3	-1	0	-1	-3	-2	-2	1	4

Is it Really Necessary?

Code	Häufig- keit	Mutier- barkeit
L	0.091	54
A	0.077	100
G	0.074	50
S	0.069	117
V	0.066	98
E	0.062	77
K	0.059	72
T	0.059	107
I	0.053	103
D	0.052	86
P	0.051	58
R	0.051	83
N	0.043	104
Q	0.041	84
F	0.040	51
Y	0.032	50
M	0.024	93
H	0.023	91
C	0.020	44
W	0.014	25

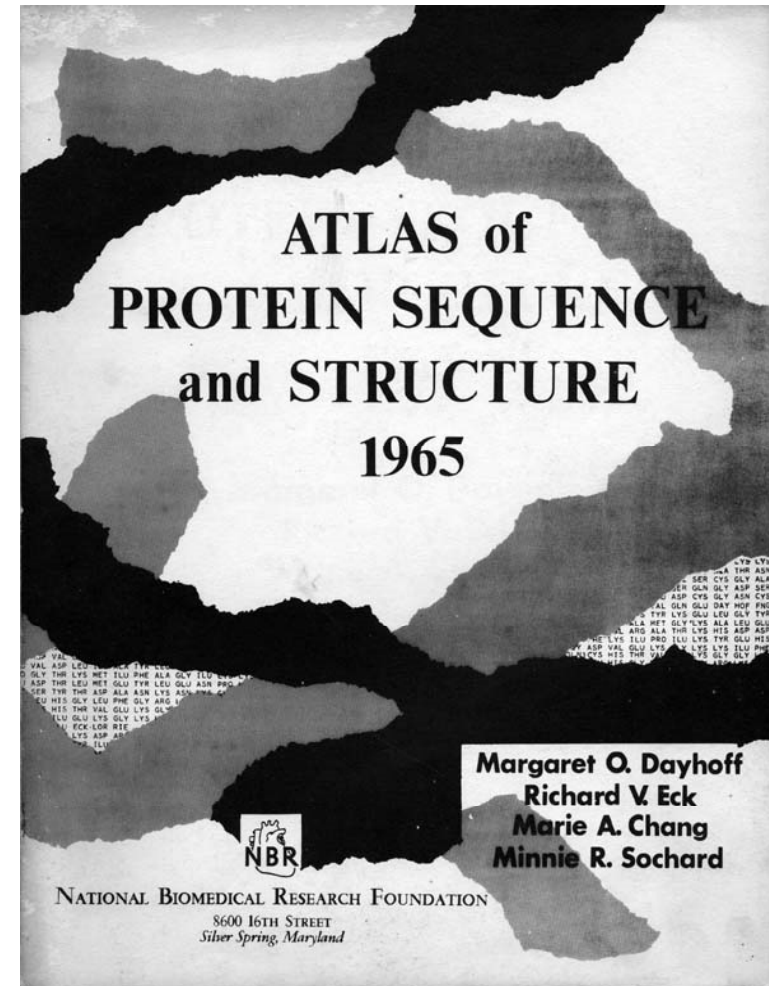
- We count how often a particular AA was **replaced by any other AA**
 - Using “sure” sequence alignments
- Replacement rate of Alanin (A) := 100%
- **Obviously no equal distribution**
- Even if we assume that mutations happen more or the less at the same rate
- ... they obviously don't survive with the same probability
 - **Mutations are suppressed** to different degrees
 - W (Tryptophan): Strong suppression
 - S (Serin): Little suppression (less than A)

Filling a Substitution Matrix

- We need **app. 200 values**
 - Assuming a symmetric matrix
- Possibility 1: Analytical
 - Capture weight, polarity, size, ...
 - Find a scoring scheme to measure the difference between two AA
 - Needs to produce a single value per AA pair
 - Not used in practice
- Possibility 2: Empirical
 - Count which substitutions survived at which frequency **in reality**
 - Needs true alignments: **Pairs of homologues and aligned sequences**

Margaret O. Dayhoff

- Goal: “Deduce evolutionary relationships of the biological kingdoms, phyla, and other taxa from sequence evidence”
- Collection of all **known protein sequences**
 - First edition: 65 proteins
 - Several releases followed
 - Resulted in the Protein Information Resource (PIR)



Thanks to Antje Krause

PAM: Point-Accepted Mutations

- Dayhoff, M. O., R. V. Eck, C. M. Park. (1972)
A model of evolutionary change in proteins.
in M. O. Dayhoff (ed.), Atlas of Protein Sequence and Structure Vol. 5.
- PAM has two meanings
 - 1 PAM – **Unit** for measuring the similarity of two AA sequences
 - PAM-X matrix – **Substitution matrix** to use when aligning two sequences that are X PAM distant

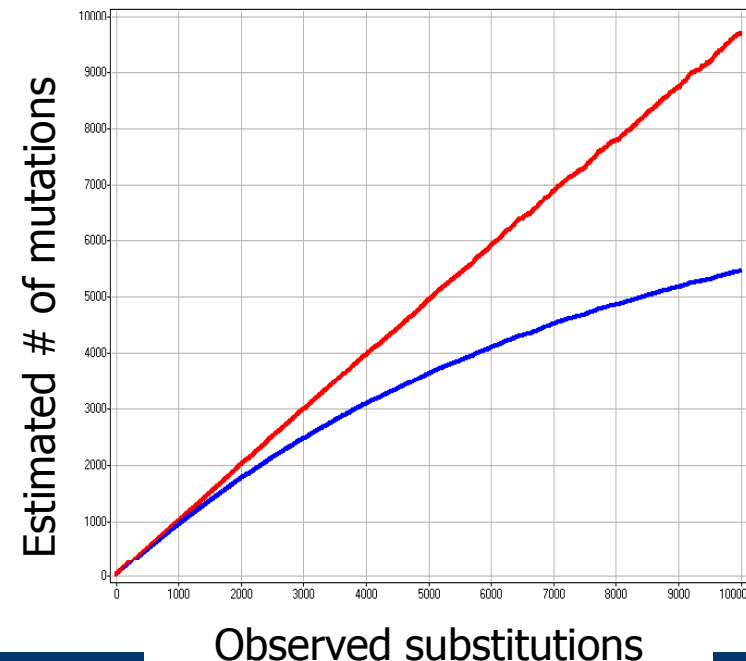
PAM as Distance Measure

- Definition

Let S_1, S_2 be two protein sequences with $|S_1|=|S_2|$. We say S_1 and S_2 are *x PAM distant*, iff. S_1 most probably was produced from S_2 with *x mutations per 100 AAs*

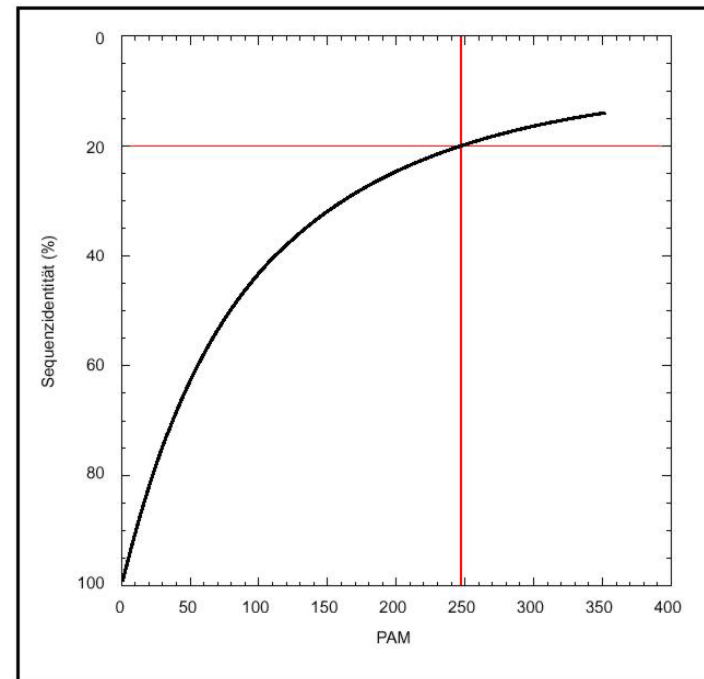
- Remarks

- PAM is motivated by evolution
- Assumptions: Mutations happen with the same rate at every position of a sequence
- If mutation rate is high, mutations will occur again and again at the same position
- PAM \neq %-sequence-identity



PAM as Distance Measure

- No INSDELS, only replacements
- The PAM distance d of two sequences **can be derived analytically** from their %-sequence-similarity p
 - $d = -3/4 * \ln(1 - 4/3 * p)$ (Jukes-Cantor model)
 - Derivation skipped
- Pairs with PAM >250 are probably not homologues
 - For those pairs,
%-sequence-identity < 20%
 - **Twilight zone**
 - Which %-sequence-identity will two random protein sequences have?



Generating a PAM Matrix

- The PAM-X matrix contains measures for the probability that a given AA was replaced by another given AA in two sequences that are x PAM distant
- Estimated from data
 - Let $(S_{1,1}, S_{2,1}), \dots, (S_{1,n}, S_{2,n})$ by **n pairs of aligned sequences**
 - Each x PAM distant
 - Compute $f(i)$, the relative frequency of AA A_i in all pairs
 - Compute **$f(i,j)$, the relative replacement frequency** from A_i to A_j
 - Number of positions k in any of the aligned pairs with $S_{1,z}[k]=A_i$ and $S_{2,z}[k]=A_j$ or vice versa
 - Then

$$M_x(i, j) = \log \left(\frac{f(i, j)}{f(i) * f(j)} \right)$$

Some Explanations

- Again: **A log-likelihood** ratio, combining
 - Expectation: chances to generate this mutation by chance given the relative frequencies of the two involved AAs
 - Observation: observed frequency of this mutation

$$M_x(i, j) = \log \left(\frac{f(i, j)}{f(i) * f(j)} \right)$$

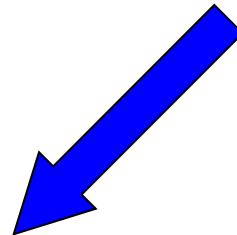
- Meaning
 - $M(i, j) = 0$: **No selection**
 - $M(i, j) < 0$: **Negative selection**, suppression of mutation
 - $M(i, j) > 0$: **Positive selection**, mutation is favored

Example

$S_{1,1}$: ACGGTGAC
 $S_{2,1}$: AGG_TGCC
 $S_{1,2}$: GTT_AGCTA
 $S_{2,2}$: TTTCAG_TA
 $S_{1,3}$: GGTCAA
 $S_{2,3}$: AGTC_A

Relative frequencies

A: 11/42	C: 8/42	G: 12/42	T: 11/42
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Mutation rates

	A	C	G	T
A	4/19	1/19	1/19	0/19
C		2/19	1/19	0/19
G			4/19	1/19
T				5/19



Matrix

	A	C	G	T
A	0,48	0,02	-0,15	-
C		0,46	-0,01	-
G			0,41	-0,15
T				0,58

Problems

- Depends on predefined alignments
- We need a substitution matrix to find optimal alignments
 - A hen-egg problem
 - Alternative: Do it manually using experience, 3D-structure, ..
- Makes several assumptions
 - Mutations are equally likely at every position in a sequence
 - Mutations are equally likely independent from AA neighbors

Real Substitution Matrices

- PAM requires large n for each x to adequately capture rare mutations
- Cure: Molecular clock assumption
 - Assume that mutations appear with equal rate over time
 - Then the frequencies of PAM- x mutations depend linearly on the frequencies of PAM-1 mutations
 - PAM- x matrices are computed by repeated matrix multiplication of PAM-1 with itself
- Complete procedure
 - Choose set of n pairs with small distance and align manually
 - Use these alignments to compute M_1
 - Compute $M_x = (M_1)^x$

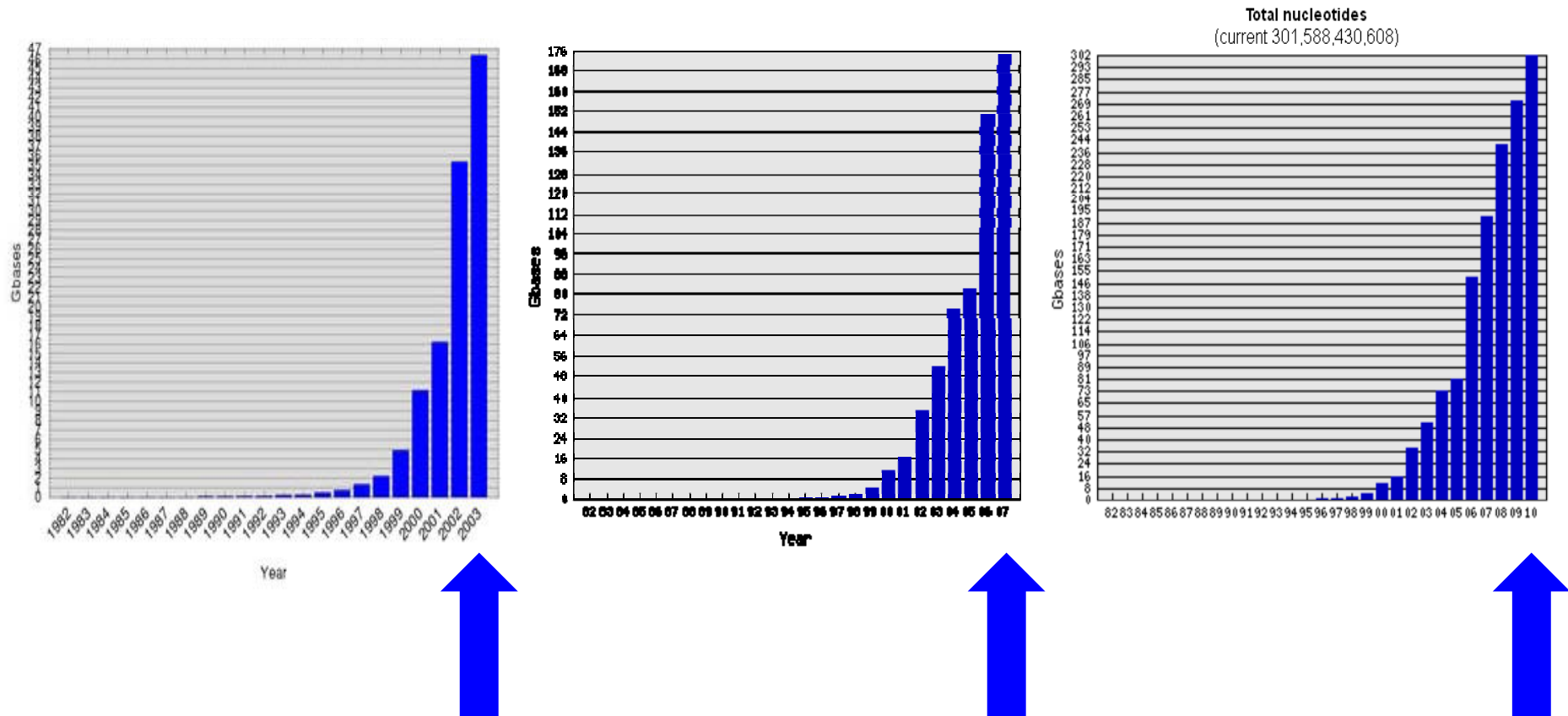
BLOSUM

- PAM is a bit old-fashioned
- **BLOSUM: BLOcks SUbstitution Matrix**
 - Henikoff and Henikoff, 1993
 - Removes assumption of equal mutation rates across each sequence position by considering **conserved blocks**
 - Direct estimation for different PAM distances instead of error-propagating self multiplication

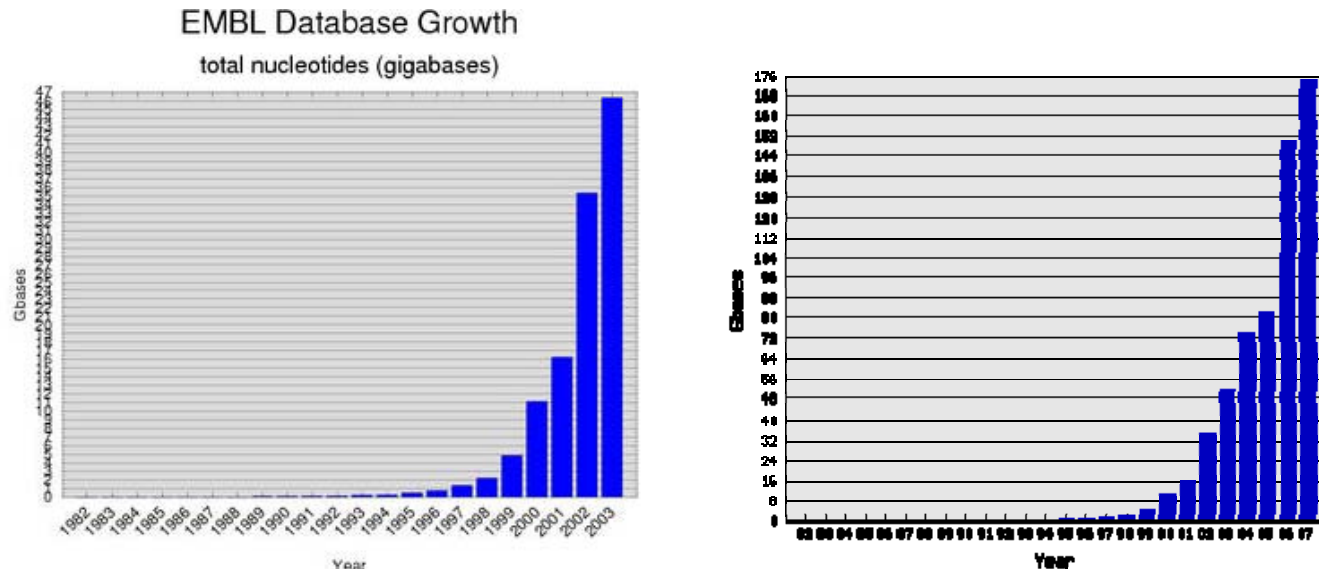
This Lecture

- Substitution Matrices
 - PAM distance
 - PAM matrices
- Scaling up Local Alignments
 - BLAST

Growth of EMBL



Scaling Up Local Alignment



- Searching similar sequences (with a high **local alignment** score) is a fundamental operation in Bioinformatics
- Sequence databases **grow exponentially**
- Naïve method does not scale at all
- We need **faster algorithms**, even if they sometimes fail

Similarity Search Problems and their Accuracy

- Task: Given a sequence s and a database D , find all sequences T in D that are sufficiently similar to s
 - Often, exactly computing T is not feasible and not necessary (think of the WWW)
 - Assume a method that finds a set X of answers for s
- How good is this method?
 - Some sequences will be in X and T – true positives
 - Some will be in X but not T – false positives
 - Some will be in T but not X – false negatives
 - Some will be neither in X nor T – true negatives

		Reality	
		+	-
Prediction	+	TruePositive (TP)	FalsePositive (FP)
	-	FalseNegative (FN)	TrueNegative (TN)

Precision and Recall

- **Precision** = $TP/(TP+FP)$

- What is the fraction of correct answers in X?
- Related to specificity

Prediction

		Reality	
		+	-
Prediction	+	TruePositive (TP)	FalsePositive (FP)
	-	FalseNegative (FN)	TrueNegative (TN)

- **Recall** = $TP/(TP+FN)$

- Which fraction of correct answers from T are also in X?
- Also called sensitivity

- **Trade-Offs**

- Usual methods compute a **score per element** of D
- All sequences with a score above a threshold t are returned as X
- Increasing t : higher precision, lower recall
- Lowering t : lower precision, higher recall
- ... if the **score correlates with correctness** ...

Example

- Let $|DB| = 1000$, $|X|=15$, $|T|=20$, $|X \cap T|=9$

	Real: Positive	Real: Negative
Alg: Positive	TP = 9	FP = 6
Alg: Negative	FN = 11	TN = 974

- Precision = $TP/(TP+FP) = 9/15 = 60\%$
- Recall = $TP/(TP+FN) = 9/20 = 45\%$

- Assume we increase t: $|X|=10$, $|X \cap T|=7$

	Real: Positive	Real: Negative
Alg: Positive	TP = 7	FP = 3
Alg: Negative	FN = 13	

- Precision: 70%, recall = 35%

BLAST

- Altschul, Gish, Miller, Myers, Lipman: „Basic Local Alignment Search Tool“, J Mol Bio, 1990
 - A **heuristic algorithm** for sequence similarity search
 - Very fast, high recall, but not perfect
 - Very successful: You “blast” a sequence
 - NCBI runs thousands of BLAST searches every day
- A **family of tools**
 - Gapped-BLAST, PSI-BLAST, MegaBlast, BLAST-ALL, PATHBLAST, Name-BLAST, ...
 - BLAST for DNA, protein, DNA-protein, protein-DNA, ...
 - We only look at the simple DNA-DNA version
 - We skip several heuristic and domain-specific tricks

Fundamental Idea

- Fundamental idea : If two sequences have a good local alignment, then this area contains, with very high probability, **a smaller area where the match is even better** (or even exact)
 - These sub-areas are called **seeds**

```
TTGACTCGATTATAGTCGCGGATATACTATCG
CCTATCACAAGAATATAGTCCCTGATCCAGC
```

```
TTGACTC GATTATAGTCGCGGAT ATACTATCG
CCTATCACA GAATATAGTCCCTGAT CCAGC
```

```
TTGACTC GATTATAGTCGCGGAT ATACTATCG
CCTATCACA GAATATAGTCCCTGAT CCAGC
```

Algorithm

- Given query sequence s and sequence database $D=\{d_i\}$
- 1. Compute **all substrings** s_i of s of length q
 - Also called q -grams
 - How many?
- 2. Find all **approximate occurrences** of all s_i in all d_j
 - Gap-free alignment with matrix; score must be above **threshold** t
 - Hits are called **seeds** –approx. occurrences of some s_i in some d_j
- 3. **Extend seeds** to left and right in s_i and d_j until
 - [Constantly update the similarity score]
 - ... the score drops sharply
 - ... s_i or s_j ends
 - ... the score gets too bad compared to other hits found earlier

Example

$q=5$, $t=3$, Matrix: $M=+1$, $R=-1$
 $s=ACGTGATA$
 $d=GATTGACGTGACTGCTAGTGATACTATAT$



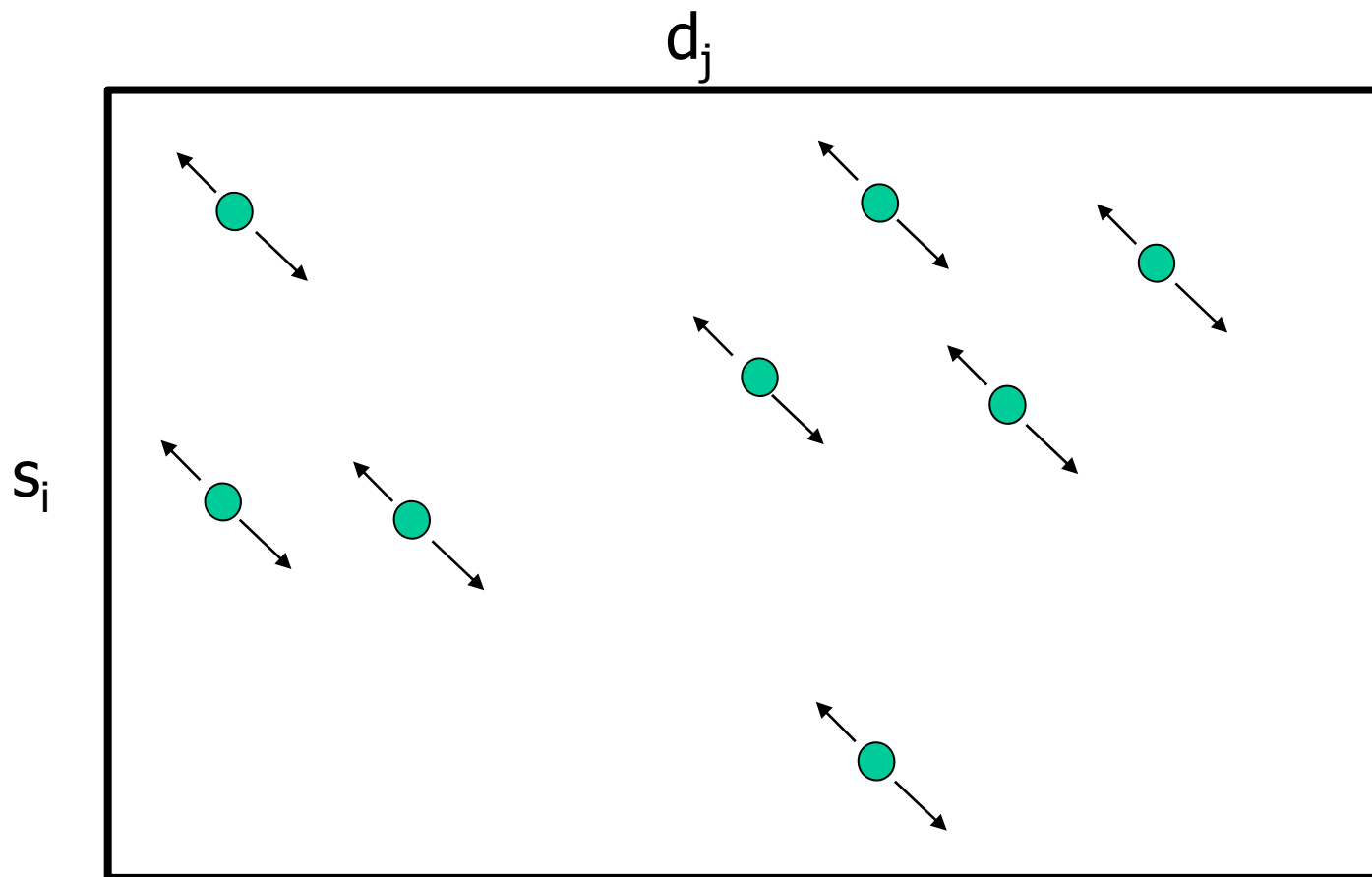
$s_1=ACGTG$
 $s_2=CGTGA$
 $s_3=GTGAT$
 $s_4=TGATA$

GATTG**ACGTG**ACTGCTAGTGATACTATAT
GATTG**ACGTGA**CTGCTAGTGATACTATAT
GATTGACGTGACTGCTAG**TGATA**CTATAT
GATTGACGTGACT**TGCTA**GTGATACTATAT



GATTG ACGTGA CTGCAAGTGATACTATAT	
ACGTGATA	5
ACGTGATA	$5+1=6$
ACGTG A T	$6-1=5$
...	...

Visualization



Properties

- Finding **seeds efficiently** requires more work
 - Pre-compute all q-grams of all d_i
 - Group by q-gram
 - Called a **hash-index** (should be kept in main memory)
 - Lookup: Given s , find all matching q-grams (as seeds)
- **Exclusion method**
 - Vast majority of all sequences in DB are never looked at because they do not contain a seed
 - The “seed” idea is the basis of nearly all fast alignment methods
- Where it fails
 - **Sensitive to t** : Too high – missing hits; too low – slow
 - Does not consider gaps

Speed – Precision - Recall

- Increasing t
 - Higher requirements for any seed
 - Less seeds, less extensions
 - Lower recall, higher speed, precision stays
- Increasing q (and adapting t)
 - Higher requirements for any seed
 - Less seeds, less extensions
 - Lower recall, higher speed, precision stays

BLAST Screenshots

The image shows two overlapping browser windows from Mozilla Firefox. The background window is the NCBI BLAST results page for query 'gi|124806265 (3279 letters)'. It displays a list of transcripts and genomic sequences on the left, and a detailed BLAST search results table at the bottom. The foreground window is the Entrez Genome view for the same query, showing a map of the human genome with various chromosomes and a color key for BLAST scores.

NCBI BLAST Results (Background Window):

Job Title: gi|124806265 (3279 letters)

BLASTN 2.2.1

Reference: Altschul, Stephen F., et al. (1990). Basic local alignment search tool. J. Mol. Biol. 215:403-410.

Database: human genome (assembly)

Query: gi|124806265 (3279 letters)

Hit List (Transcripts):

- NM_022726.2
- NT_007933.14
- NW_923640.1
- NT_079596.2
- NT_005403.16
- NW_921618.1
- NT_011786.15
- NW_927721.1
- NT_016354.18
- NW_922217.1
- NT_007299.12
- NW_923184.1
- NT_006576.15
- NW_922562.1
- NW_922162.1
- NT_011903.12
- NT_011630.14
- NT_025028.13
- NT_077661.2
- NT_026437.11
- NT_113923.1
- NT_023666.17
- NT_007819.16
- NT_025741.14
- NT_022171.14
- NT_026970.9
- NW_927756.1
- NW_927710.1
- NW_927106.1
- NW_925918.1
- NW_925561.1
- NW_923907.1
- NW_923240.1
- NT_079592.2
- NW_923095.1
- NW_921585.1

Entrez Genome View (Foreground Window):

NCBI Map Viewer

Search for: on chromosome(s): assembly: All

Homo sapiens (human) genome view

Build 36.2 statistics [Switch to previous build](#)

BLAST search the human genome

Hit GIS: 1 2 3 4 5 6 7 8 9 10 11 12 13

Hits: 3 9 31 4 11 16 30 26 3 14 6 8 2 2 8

Hit GIS: 14 15 16 17 18 19 20 21 22 X Y III not placed

Hits: 2 4 8 2 7 3 4 1 2 11 2 4 2

Color key for scores: < 40 40-50 50-80 80-200 >= 200

Back to BLAST alignments page

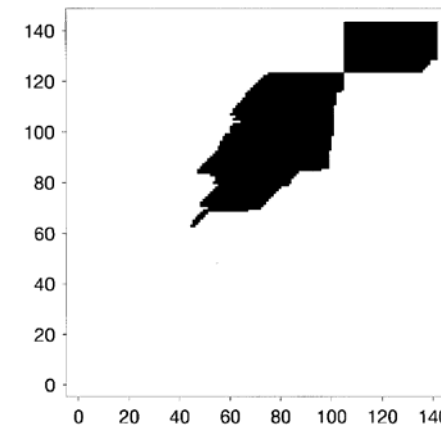
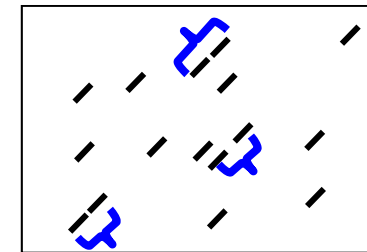
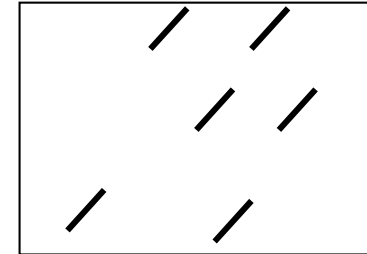
BLAST search results: 100 BLAST hits found

Query: gi|124806265[ref]XM_001350639.1 Plasmodium falciparum 3D7 hypothetical protein, conserved (PFL1345c) mRNA, complete cds

Chr	Assembly	Map element	Type	BLAST results
				Hits Score E value
1	reference	NT_032977	CONTIG	2 42.8 2.6
1	Celera	all matches		

BLAST-2

- Altschul, Madden, Schaffer, Zhang, Zhang, Miller, Lipman: „Gapped BLAST and PSI-BLAST: a new generation of protein database search programs“, NAR, 1997
- Faster
 - BLAST: 90% of time spend in extensions
 - BLAST2: **Two seeds** in short distance
 - Needs a decrease in t
- Higher recall
 - BLAST didn't even consider gaps in the extension phase
 - BLAST2: **Full local alignment** starting from seeds
 - Allows an increase of t



Further Reading

- Substitution matrixes: Krane & Raymer, Chapter 3
- BLAST, BLAST2: Merkl & Waack, Chapter 12