Datenbanksysteme II: Implementation of Database Systems

Multidimensional Indexing

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Material von

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Content of this Lecture

- Introduction to multidimensional indexing
- Partitioned Hashing
- Grid files
- kdb Trees
- R trees

Multidimensional Indexing

- Access methods so far
  - Support access on attribute(s) for
    - Point query: Attribute = const (Hashing and B-Tree)
    - Range query: $\text{const}_1 \leq$ Attribute $\leq \text{const}_2$ (B-Tree)
- More complex queries
  - Point query on more than one attribute
    - Combined through AND (intersection) or OR (union)
  - Range query on more than one attribute
  - Queries for objects with size
    - "Sale" is a point in a multidimensional space
      - Time, location, product, ...
    - Geometric objects have size: rectangle, cubes, polygons, ...

Geometric Objects

- GIS (geographic information system) store rectangles

  ```
  RECT (X1, Y1, X2, Y2)
  ```

  (X1 ,Y1) lower left corner and (X2 ,Y2) upper right corner
- Queries
  - Box query: All rectangles containing point (5,6)

    ```
    SELECT * FROM RECT
    WHERE  X1 ≤ 5 and Y1 ≤ 6 and
           X2 ≥ 5 and Y2 ≥ 6
    ```

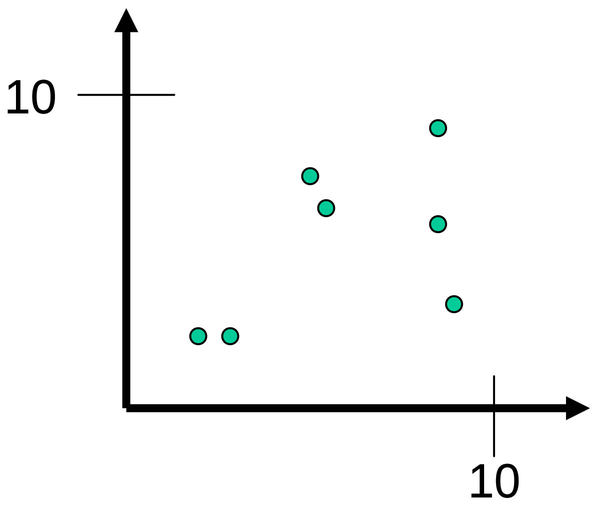
    - Similar to range query – all points within a given rectangle
  - Partial match query: Rectangles containing points with X=3

    ```
    SELECT * FROM RECT
    WHERE  X1 ≤ 3 and X2 ≥ 3
    ```

  - All rectangles with non-empty intersection with rectangle Q

    ```
    SELECT * FROM RECT
    WHERE  …
    ```

Composite Indexes

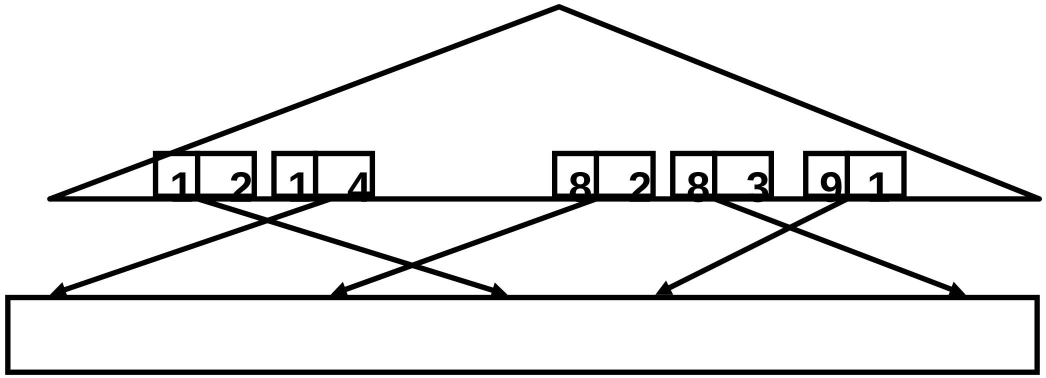


Point	X	Y
P1	2	2
P2	2,5	2
P3	4,5	7
P4	4,7	6,5
P5	8	6
P6	8	9
P7	8,3	3

- Imagine composite index on (X, Y)
- Box queries: efficiently supported
- Partial match query
  - All points/rectangles with X coordinate between ...
    - Efficiently supported
  - All points/rectangles with Y coordinate between ...
    - Not efficiently supported

Composite Index

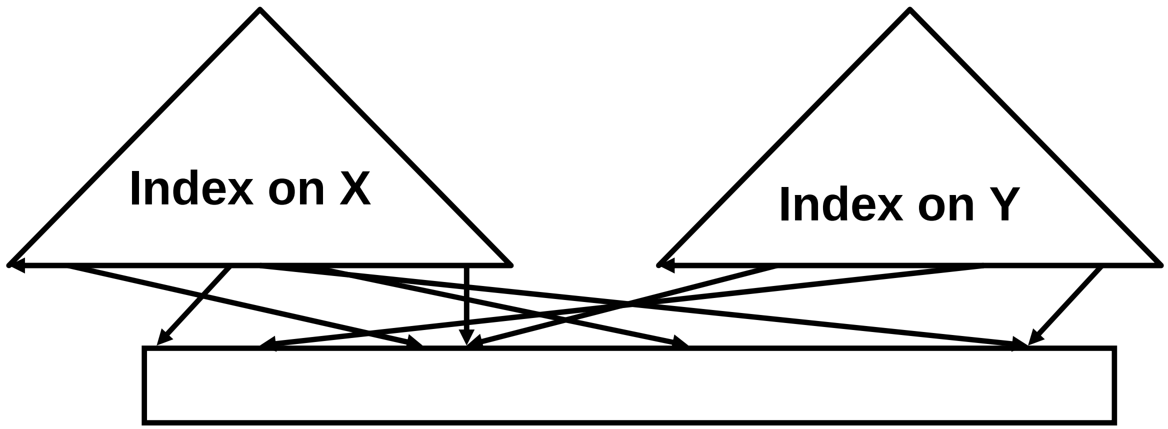
- One index with two attributes (X, Y)



- General
  - Prefix of attribute list in index must be present in query
  - The longer the prefix in a query, the more efficient
- Alternatives
  - Also index (Y, X)
    - Combinatorial explosion for more than 2 attributes
  - Use independent indexes on each attribute

Independent Indexes

- One index per attribute



- Partial match query on one attribute: supported
- Partial match query on many attributes or box query
  – Compute TID lists for each attribute
  – Intersect

Example – Independent versus Composite Index

- Consider 3 dimensions of range 1,…,100
  - 1.000.000 points, uniformly (randomly) distributed
  - Index blocks holding 50 keys or records
  - Index on each attribute has height 4
- Find points with 40≤x≤50, 40≤y≤50, 40≤z≤50
  - Using x-index, we generate list |X|~100.000
  - Using y-index, we generate list |Y|~100.000
  - Using z-index, we generate list |Z|~100.000
  - For each index, we have 4+100.000/50=2004 IO
    - TIDs are sorted in sequential blocks, each holding 50 TIDs
  - Hopefully, we can keep the three lists in main memory
  - Intersection yields app. 1.000 points with 6012 IO
    - Why 1000 points??
- Using composite index (X,Y,Z)
  - Number of indexed points doesn't change
  - Key length increases – assume blocks hold only 30 (10) keys or records
  - Index has height 5 (6)
    - This is worst case – index blocks only 50% filled
  - Total: 5 (6) +1000/30 (10) ~ 38 IO (104)

Generalization

- Assume d dimensions, n records, k keys
- Assume query selectivity in each dimension s
- Independent indexes
  - Each independent index has height $\log_k(n)$
  - We find s*n TIDs in (s*n)/k blocks
  - All together: $C_1 = d*(\log_k(n)+(s*n)/k)$
- Composite index
  - Index has height $\log_r(n)$ for some r<k
  - We find s^d*n TIDs in $(s^d*n)/r$ blocks
  - All together: $C_2 = \log_r(n)+ (s^d*n)/r$
- For d=5, n=1.000.000, k=50, r=30, s=0.1
  - C_1= 20+10000, C_2= 4+0
    - On average, the result will already be empty
- For d=8, n=1E9, k=50, r=10, s=0.01
  - C_1= 48+1.600.000, C_2=9+0

Conclusion 1

- We want composite indexes
  - Much less IO
  - Things get worse for bigger d
    - TID lists don't fit into main memory – paging, more IO
    - Reading large TIDs list again and again is more work than scanning relation once
    - Linear scanning of relation might be faster
  - Advantage grows "exponentially" with number of dimensions and selectivity of predicates
  - Things get complicated if data is not uniformly distributed
    - Dependent attributes (age – weight, income, height, ...)
    - Clustering of points
    - Histograms (later more)

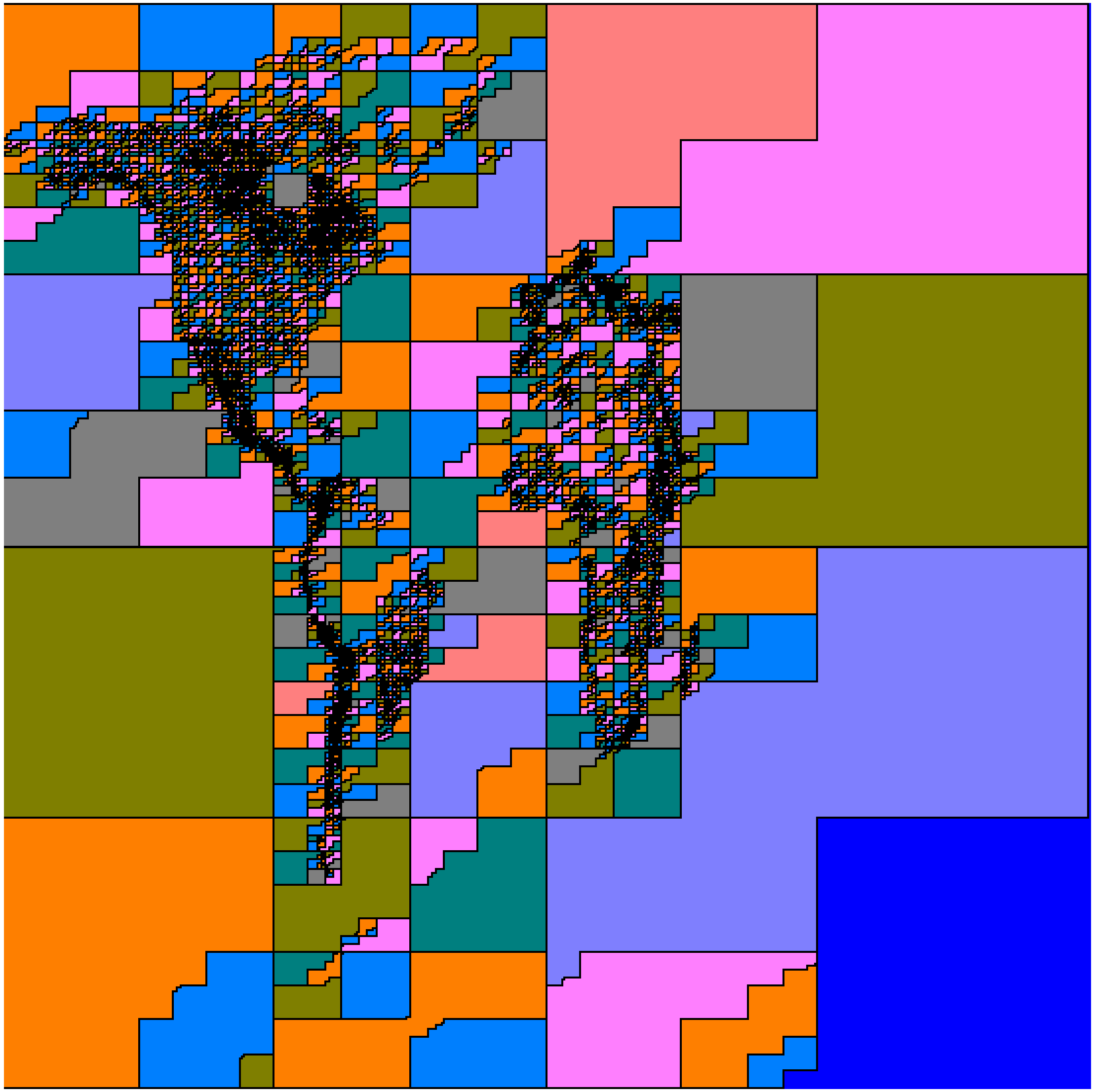
Conclusion 2

- But: To support partial match queries, we need to index all combinations
  - Impossible
- Solution: Use multidimensional indexes
  - General: Improvement, but no solution
  - “Curse of dimensionality” still valid
    - Most md indexes somehow degrade for many dimensions
    - Trees difficult to balance, very bad space usage, excessive management cost, expensive insertions/deletions, ...
  - Commercial databases use bitmap indexes
    - Very small memory footprint
  - Multidimensional indexes are used for geometric objects
    - Oracle has R tree in spatial extender

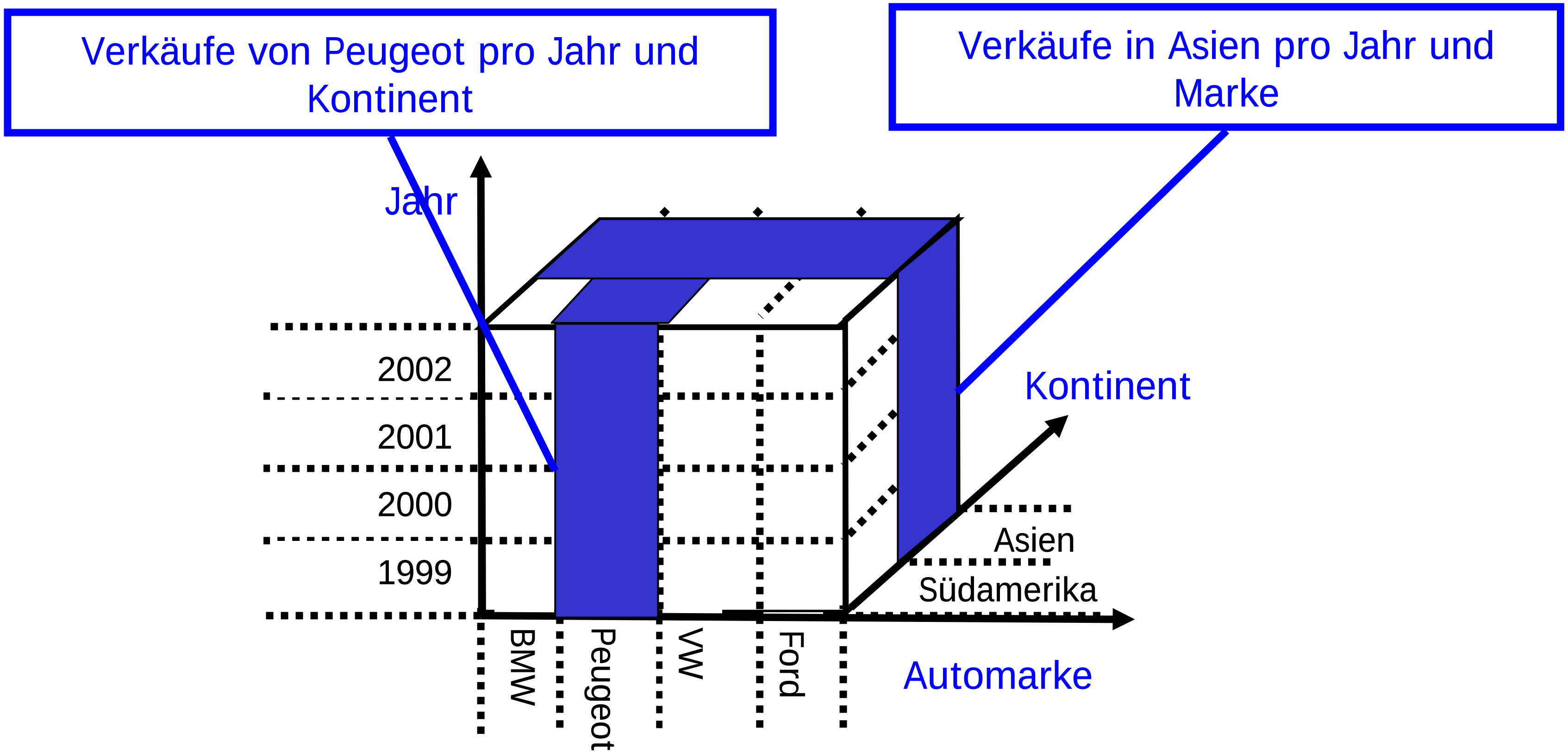
Multidimensional Indexes

- All dimensions are equally important
- Neighbors in space are (hopefully) stored on nearby blocks
  - That is the clue
  - Difficult to achieve
- Supported types of objects
  - With size
  - Without size (points)
- Supported queries
  - Exact match point queries
  - Partial match point queries
  - Box queries (range queries)
  - Nearest neighbor queries
    - In multidimensional space

Geographic Information Systems

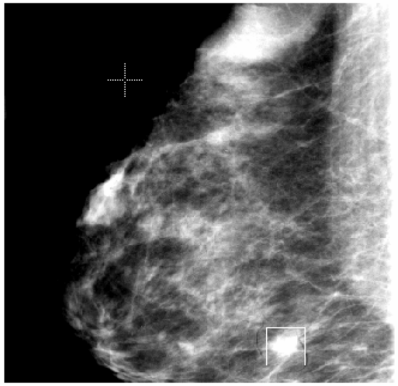
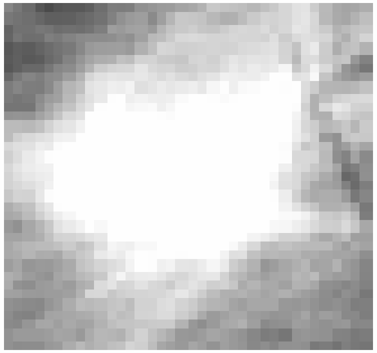
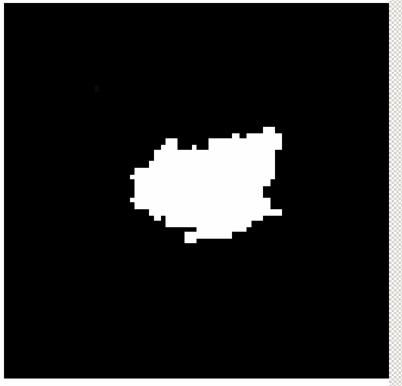


Data Warehousing



- More dimensions: customer, logistic centre, supplier, company division, ...

Multimedia Databases

- Map object into feature vector
  - Here: Tumor images
  - Feature vector are derived from mathematical morphology
    - Can be computed in varying granularity (different length of vector vectors)
    - Filling / bordering picture using differently coarse brushes
- Compute nearest neighborhood queries in feature space
  - Filters away most false positives
  - Usually, final costly check on real object still necessary (but on few)

Content of this Lecture

- Introduction to multidimensional indexing
- Partitioned Hashing
- Grid files
- kdb Trees
- R trees

Partitioned Hashing

- Partitioned Hashing
  - Let A_1 , A_2 ,…, A_k be search keys
  - Define a hash function for each A_i ; interpret result as bit string
  - Global hash key: concatenation of the attribute bit strings
  - Definition
    - Let $h(A_i)$ map each A_i into a integer with b_i bit
    - Let $b=\sum b_i$ (length of global hash key in bits)
    - The global hash function

$$h\ (v_1\ ,\ v_2\ ,\ .\ .\ .\ .\ ,\ v_k\ )\ \rightarrow [0,\ \ldots,\ 2^b-1]$$

is defined as

$$h\ (v_1\ ,\ v_2\ ,\ .\ .\ .\ .\ ,\ v_k\ )\ = h_1\ (v_1) \oplus h_2\ (v_2) \oplus \ldots \oplus h_k\ (v_k\ )$$

    - We need $B = 2^b$ buckets
      - Static address space – dynamic structures later

Example

- We want to store points
  - (3,6), (6,7), (1,1), (3,1), (5,6), (4,3), (5,0), (6,1), (0,4), (7,2)
- Let hash function h_1, h_2 be

$$h_i\ (v_i) = \begin{cases} 0 & \text{if } 0 \leq v_i \leq 3 \\ 1 & \text{otherwise} \end{cases}$$

- Thus, there are 4 buckets with address 00, 01, 10, 11

v_2

	0	1
0	**(1,1) (3,1)**	**(3,6) (0,4)**
1	**(4,3) (5,0) (6,1) (7,2)**	**(6,7) (5,6)**

v_1

Queries with Partitioned Hashing

- Exact point queries
  - Direct access to bucket possible
- Partial match queries
  - Only parts of the global hash key are determined
  - Use those as filter; scan all buckets passing the filter
  - Let $c = \sum b_i$ be the number of unspecified bits
    - Then 2^c buckets must be searched
    - These are certainly not ordered (ordered on what?) – random IO
- Range queries
  - Not supported, if hash function doesn't preserve order
  - Example of order-preserving hash function??

Order Preserving Hash Functions

- Not order preserving: modulo
- Order preserving: division
- Example
  - Suppose 3 dimensions, each with range 1..1024 (10 bits)
  - Use 3 highest bits as hash key in each dimension
    - Equal to division by 64 (right-shift 7 times)
  - Global hash key: 9 bit, hence 2^9=512 buckets
  - Partial range query: points with 200<y<300 and z<600
    - $h_y(200)=001$, $h_y(300)=010$, $h_z(600)=100$
    - Scan buckets with
      - X-coordinate: ?
      - Y-coordinate: between 001 and 010 (001, 010)
      - Z-coordinate: <100 (000, 001, 010, 011,100)
    - We need to scan 8 (x) * 2 (y) * 4 (z) = 64 buckets
- But: Very vulnerable to not-uniformly distributed data
  - Data with Gauss distribution (weight, height, age, ...) is clustered in the centre of each dimension
  - Use Modulo instead – and lose order-preservation

Conclusions

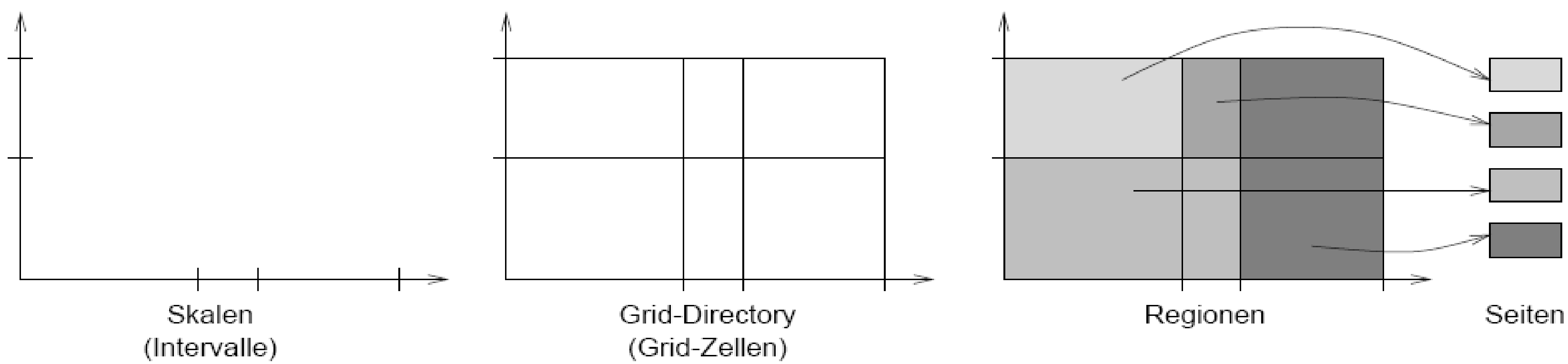
- Can only store point objects
- Has static address space as described here
  - Can be combined with extensible/linear hashing
  - Hash keys of different partitions grow/shrink independently
  - Directory in extensible hashing can grow quite large
    - Must be buffered; more IO
- No adaptation to clustered data – overflow buckets or large directories

Grid File

- Probably the most classic multidimensional index structure
  - "Quite" simple: searching, indexing, deleting
  - Good for uniformly distributed data, cannot handle skewed data well
  - Many variations (we will point to different options)
- Design goals
  - Index point objects
  - Support exact, partial match, and neighbor queries
  - Guarantee "two IO" access to each point
    - Under some assumptions
  - Do not prefer any dimension
  - Adapt dynamically to the number of points

Principle

- Partition each dimension into disjoint intervals, called scales
- The intersection of all intervals defines all grid cells
  - Convex d-dimensional hypercubes
  - Grid cells hold pointer to all data objects in that cell
  - When cell overflows – split (no overflow blocks)
  - Each point falls into exactly one grid cell
  - Grid cells are managed in the grid directory
- Grid cells are either
  - Directly addressed – each cell is one bucket = one block on disc
  - Grouped into convex, d-dimensional grid regions



Exact Point Search

- Compute grid cell coordinate
  - We keep scales for each dimension in memory
  - Looking up point coordinate in scales gives coordinates for each dimension
  - Map coordinate to block address on disk
    - Requires that grid directory on disk is organized as an array
    - Costly reorganization upon insertion and deletion – later
- Load grid directory
  - Look up block in grid directory (1st IO)
  - Find pointer to data bucket
- Access data bucket / block
  - 2nd IO
  - Search point in block

Range Query, Partial Match Query

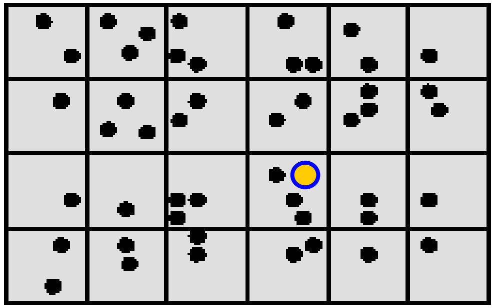
- Range query
  - Compute grid cell coordinate for each end point
  - All grid directory entries in that range may contain points
- Partial match query
  - Compute partial grid cell coordinates
  - All grid directory entries with these coordinates may contain points

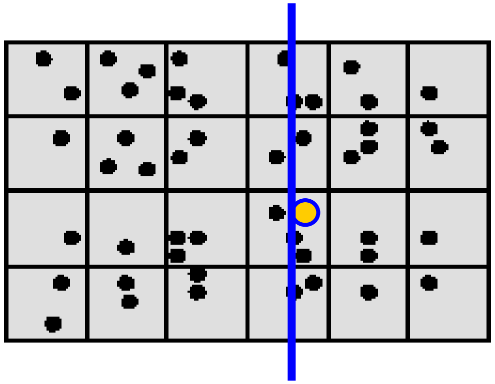
Inserting Points

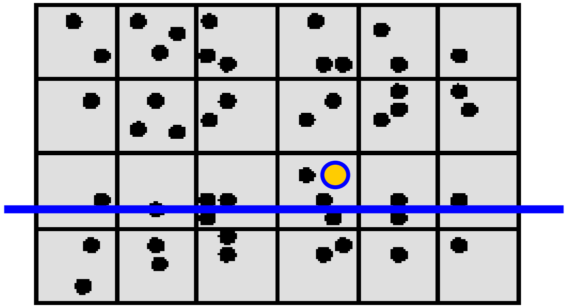
- Search grid cell
  - If data bucket has space – no problem
- Otherwise
  - Without grid regions
    - Split space
      - Choose a dimension and an interval to split
      - Split all affected grid cells
    - Consider n dimensions and d_i intervals in dimension i
      - A split in dimension (last) increases grid directory by $d_1 * d_2 * \ldots * d_{n-1}$ entries
      - Example: d=3, d_i=4
        - » Grid directory has 4^3 = 64 entries
        - » Splitting one interval generates 4^2 new entries
      - Directory blocks need to be reorganized to allow coordinate computation
    - Problem – grid directory grows very fast
    - Many empty cells (NULL pointer) or almost empty cells
    - Choice of dimension and interval is very difficult and never perfect
      - Optimally, we would like to split as many very full blocks as possible
      - This is an optimization problem in itself

Example

- Imagine one block holds 3 pointers
  - Usually we have unevenly spaced intervals
- New point causes overflow
- Where should we split?
- Vertical split
  - Splits 2 (3,4)-point blocks
  - Leaves one 3-point block
- Horizontal split
  - Splits 2 (3,4)-point blocks
  - Leaves one 3-point block
- Need to consider $O(d_i^{n-1})$ regions





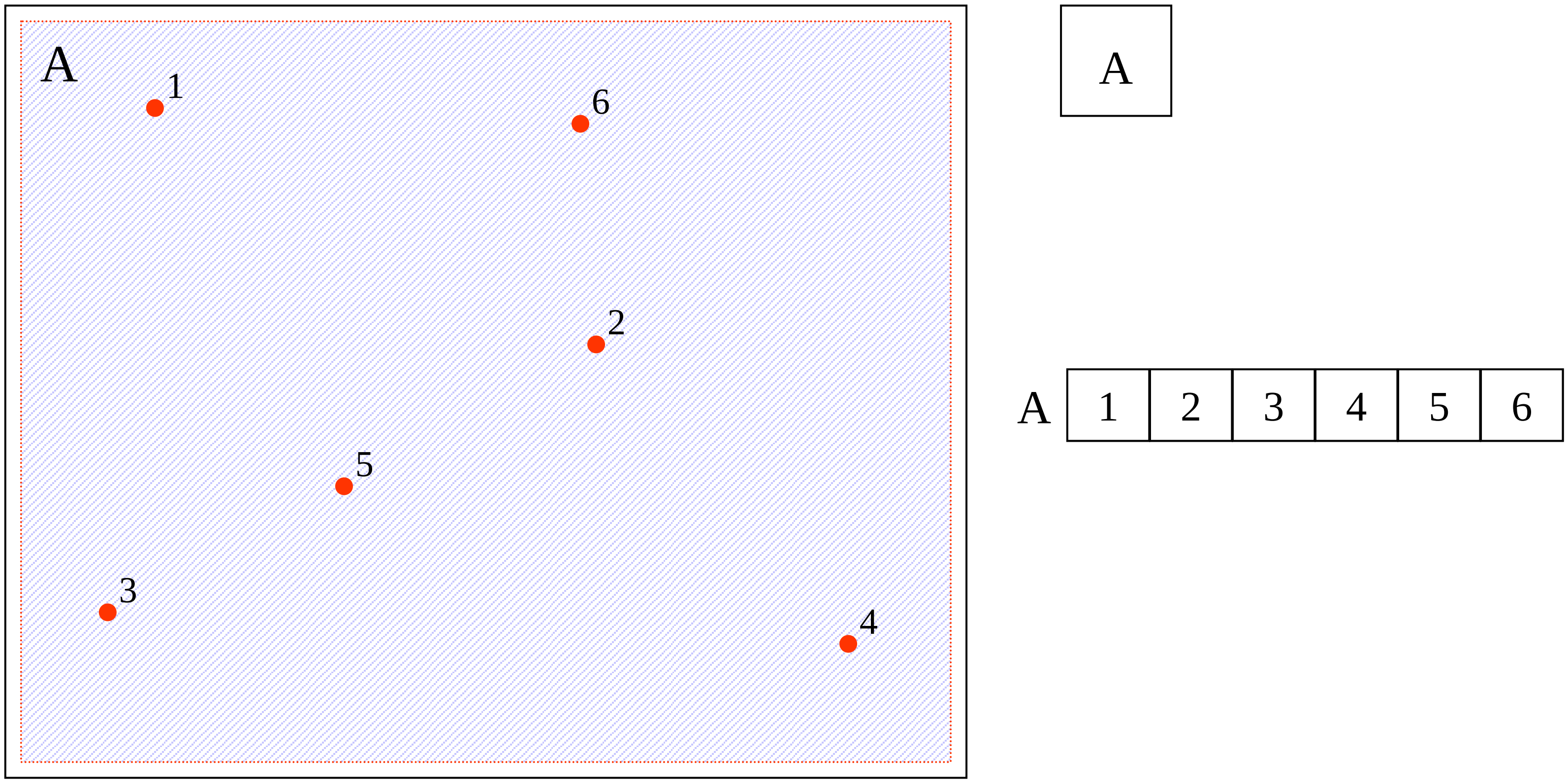


Inserting Points -2-

- With grid regions
  - Search grid region
  - Space in bucket of region?
    - No problem
  - Region coarser-grained than scales?
    - Split region into smaller regions (or cells)
    - Possible split dimensions/axes: interval borders not used for split yet
  - Region already at finest level
    - Choose split as without grid regions
    - All but the overflowed grid cell remain unchanged
      - Split is not performed; regions “raise” in granularity
    - Directory need to be extended and reorganized
- Grid regions help to prevent the “many almost empty blocks” problem

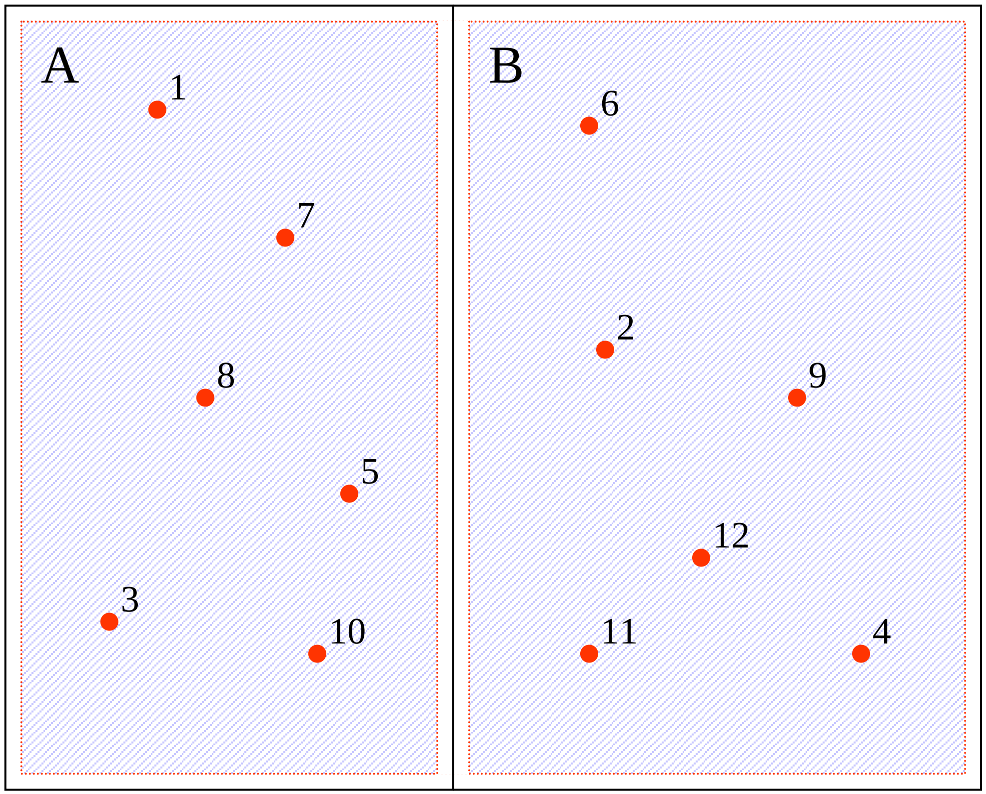
Grid File Example 1 (from Johannes Gehrke)

(N=6)



Grid File Example 2

(N=6)

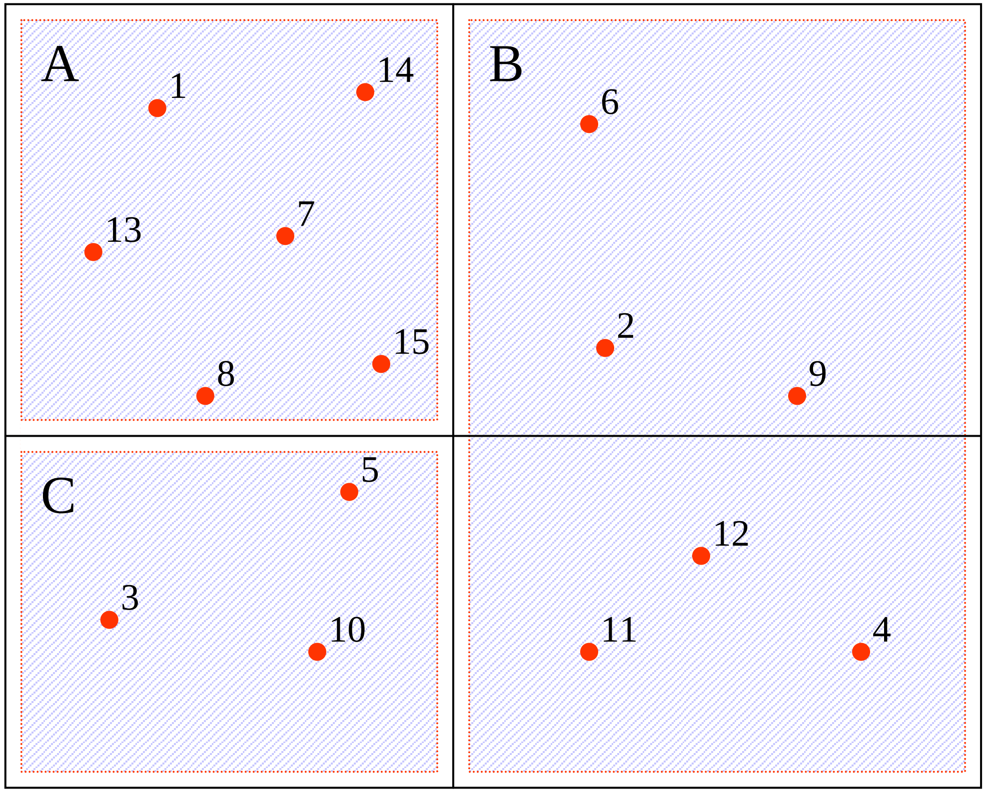


A	B

A	1	3	5	7	8	10
B	2	4	6	9	11	12

Grid File Example 3

(N=6)

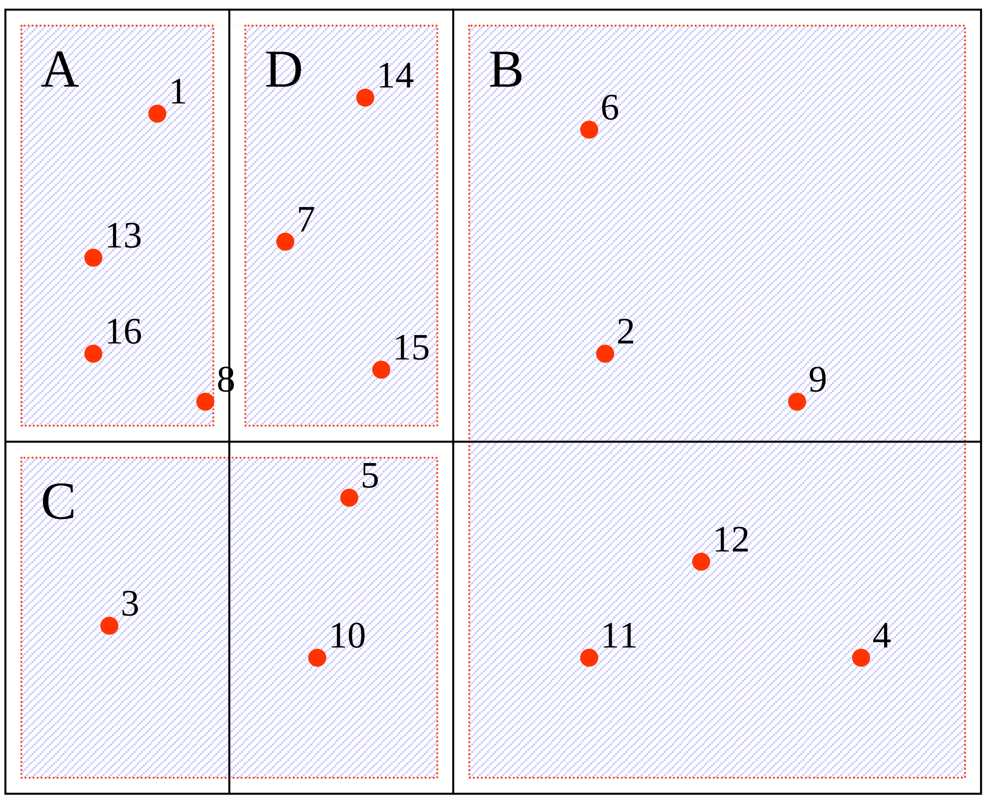


A	B
C	B

A	1	7	8	13	14	15
B	2	4	6	9	11	12
C	3	5	10			

Grid File Example 4

(N=6)

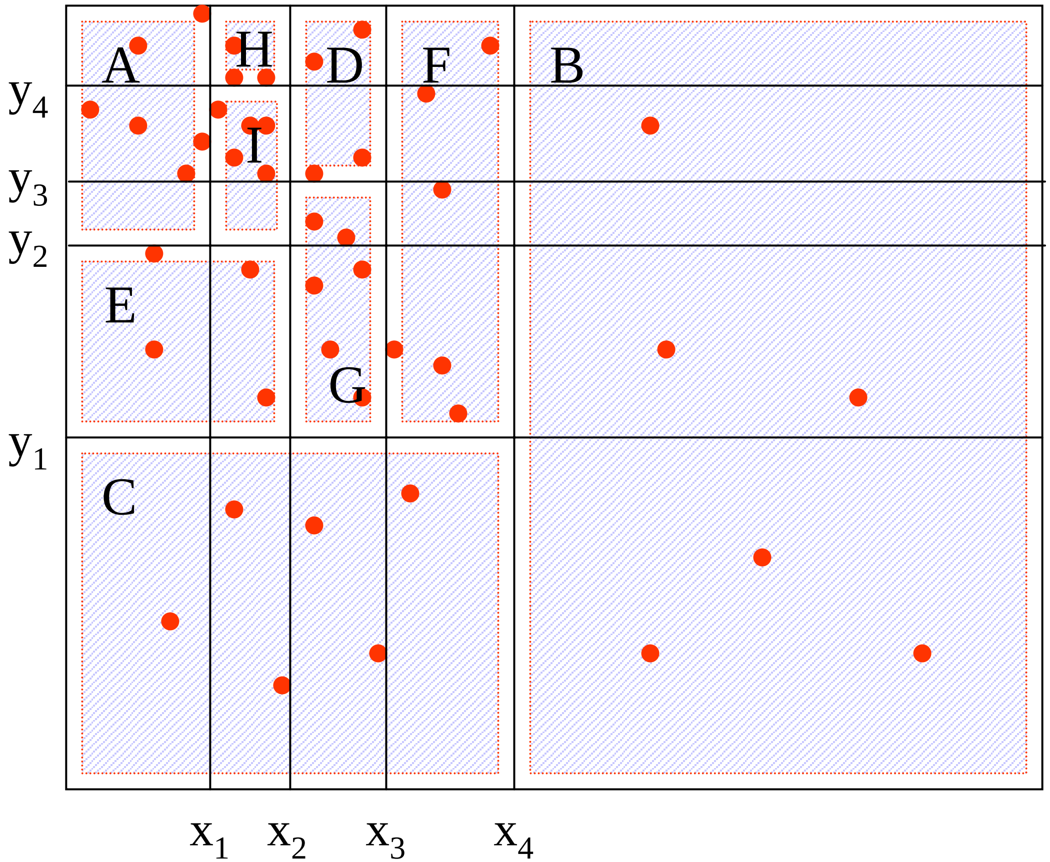


A	D	B
C	C	B

A	1	8	13	16		
B	2	4	6	9	11	12
C	3	5	10			
D	7	14	15			

Grid File Example 5

(N=6)



A	H	D	F	B
A	I	D	F	B
A	I	G	F	B
E	E	G	F	B
C	C	C	C	B

Deleting Points

- Search point and delete
- If regions become "almost" empty, choose merges
  - A merge is the removal of a split
  - Must build larger convex regions
  - This can become very difficult
    - Potentially, more than two regions need to be merged to keep convexity condition
  - Example:
    Where can we merge regions??

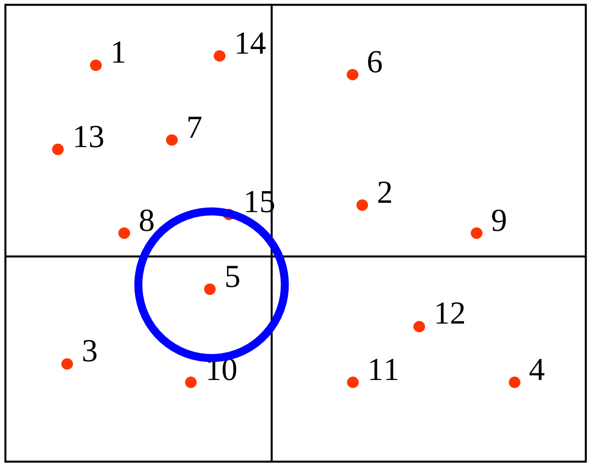
A	H	D	F	B
A	I	D	F	B
A	I	G	F	B
E	E	G	F	B
C	C	C	C	B

Nearest Neighbor Queries

- Search point
- Search points in same region and choose closest
  - If no point in same region, check surrounding buckets
  - Can we finish if point was found??

Nearest Neighbor Queries

- Search point
- Search points in same region and choose closest
  - If no point in same region, check surrounding buckets
  - Can we finish if point was found??
  - Usually not
    - Compute distance to all interval border (hyperplanes)
    - If point found is closer than all borders, we can finish



What‘s in a Bucket?

- Buckets hold “the data”
- Choices
  - Complete tuples
    - Not compatible with other database structures (indexes, etc.)
    - Few records per data blocks
    - Frequent splits, fast growing directory
  - Only TIDs
    - Many records per data block, few splits, small directory
    - But queries need to check (load) all tuples referenced in a block to check real coordinates
    - Too expensive
  - TIDs and coordinates
    - Middle way between other choices
    - Medium number of records per block, moderate size of grid directory
    - No access to tuples necessary for checking coordinates

Conclusions

- Grid files always split at hyperplanes parallel to the dimension axes
  - This is not always optimal
  - Use other than rectangles as cell structure: circles, polygons, etc.
  - More complex– forms might not disjointly fill the space any more
  - Allow overlaps - R trees
- Good: Good bucket fill degrees
  - Thanks to grid regions
- Bad: Grid directory grows very fast
- Each split decision finally becomes valid for all covering regions
  - Need not be realized at once, but restricts later choices
  - Bad adaptation to skewed data
  - The more dimensions, the worse

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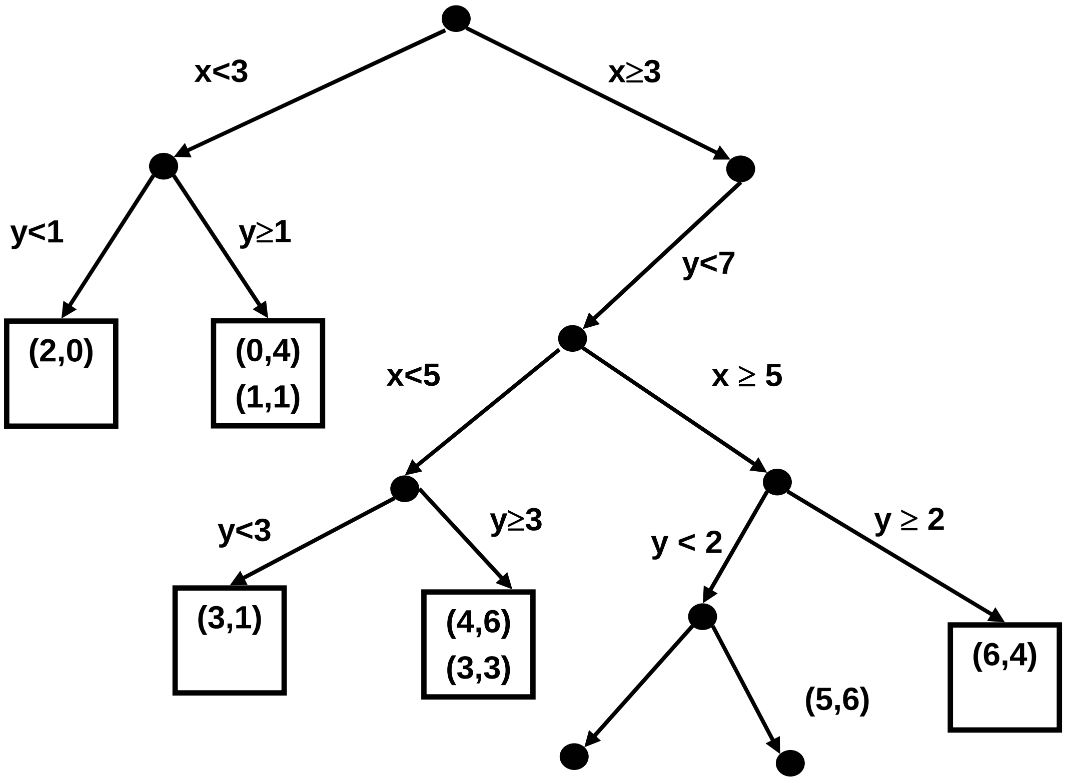
- Partitioned Hashing
- Grid files
- kd and kdb Trees
- R trees

kd-Tree

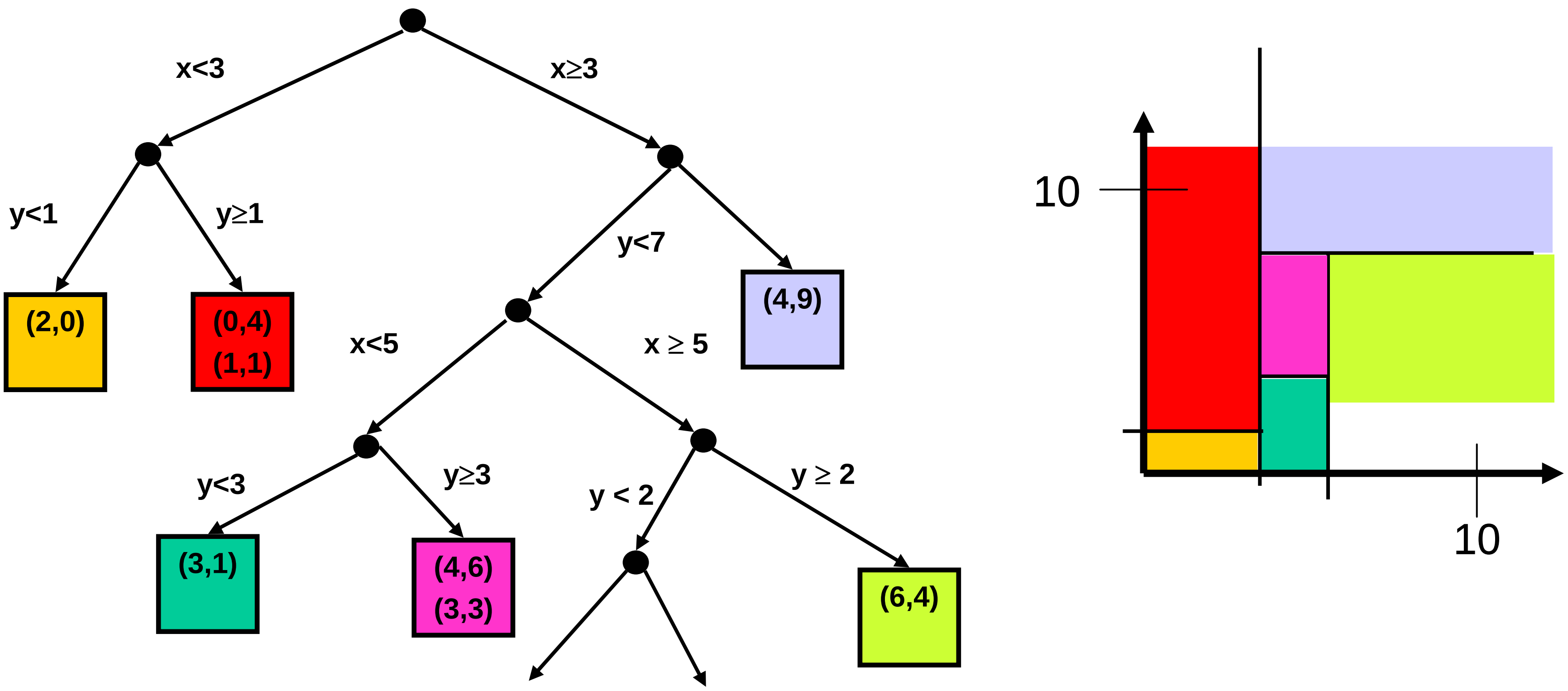
- Grid file disadvantages
  - All hyperregions of the n-dimensional space are eventually split at the same dimension/position
    - Although not all regions are actually performing the split
  - First cell that overflows determines split
  - This choice is global and never undone
- kd-trees
  - Multidimensional variation of binary search trees
  - Hierarchical splitting of space into regions
  - Regions in different subtrees may use different split positions
  - Better adaptation to clustering of data than grid files
  - kd-tree is mostly a main memory data structure
    - IO-optimization for layout of inner nodes later (kdb)

kd-Tree General Idea

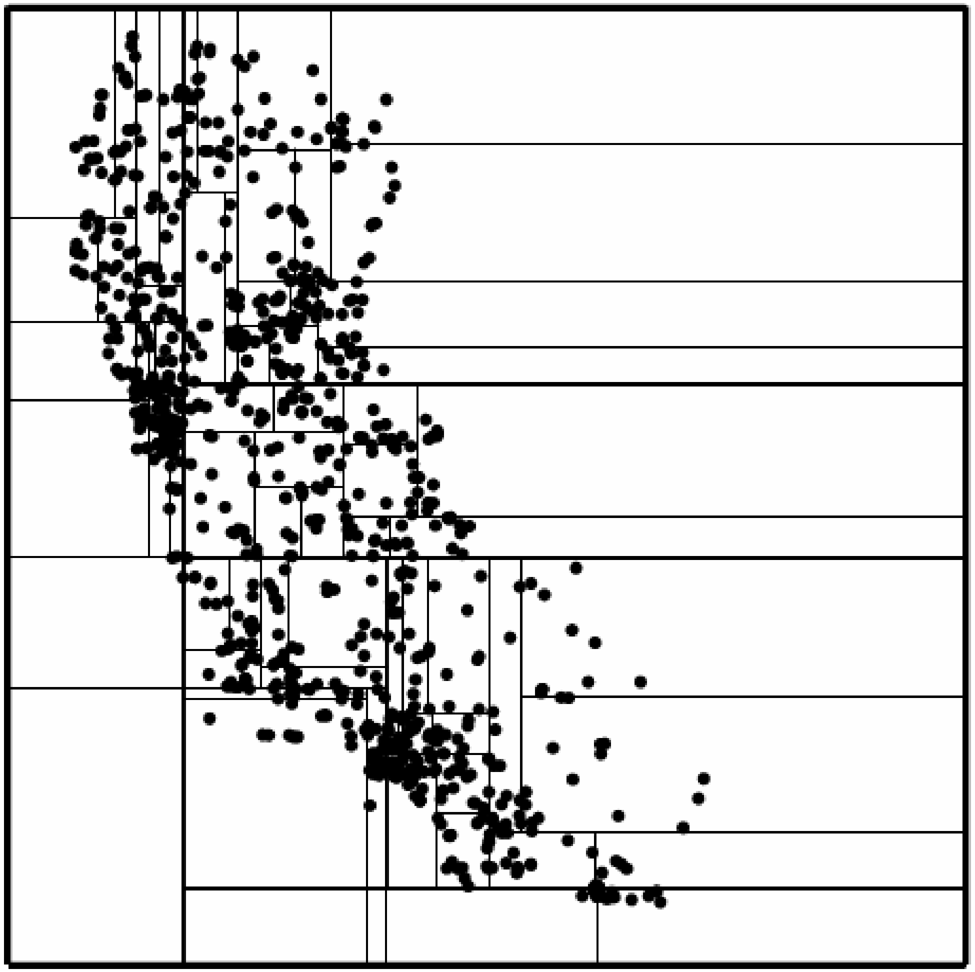
- Binary, rooted tree
- Each inner node has two children
- Path is selected based on a pair (dimension / value)
- Dimensions need not be statically assigned to levels of the tree
  - Can be rotating, random, decided at time of block split, …
  - Usually: rotating
- Data points are only stored in leaves
- Each leave stores points in a n-dimensional hypercube with m border planes (m≤n)



Example – the Brick wall



Local Adaptation



kd-Tree Search Operations

- Exact point search
  - ??
- Partial match query
  - ??
- Range query
  - ??
- Nearest Neighborhood
  - ??

kd-Tree Search Operations

- Exact point search
  - In each inner node, decide direction based on split condition
  - Search leaf for searched point
- Partial match query
  - If dimension of condition in inner node is part of the query – proceed as for exact match
  - Otherwise, follow all children in parallel
    - Leads to multiple search paths
- Range query
  - Follow all children matching the range conditions
    - Again: multiple search paths
- Nearest Neighborhood
  - Chose likely “close-enough” range and perform range query
  - If no success, iteratively broaden range

kd-Tree Insertion

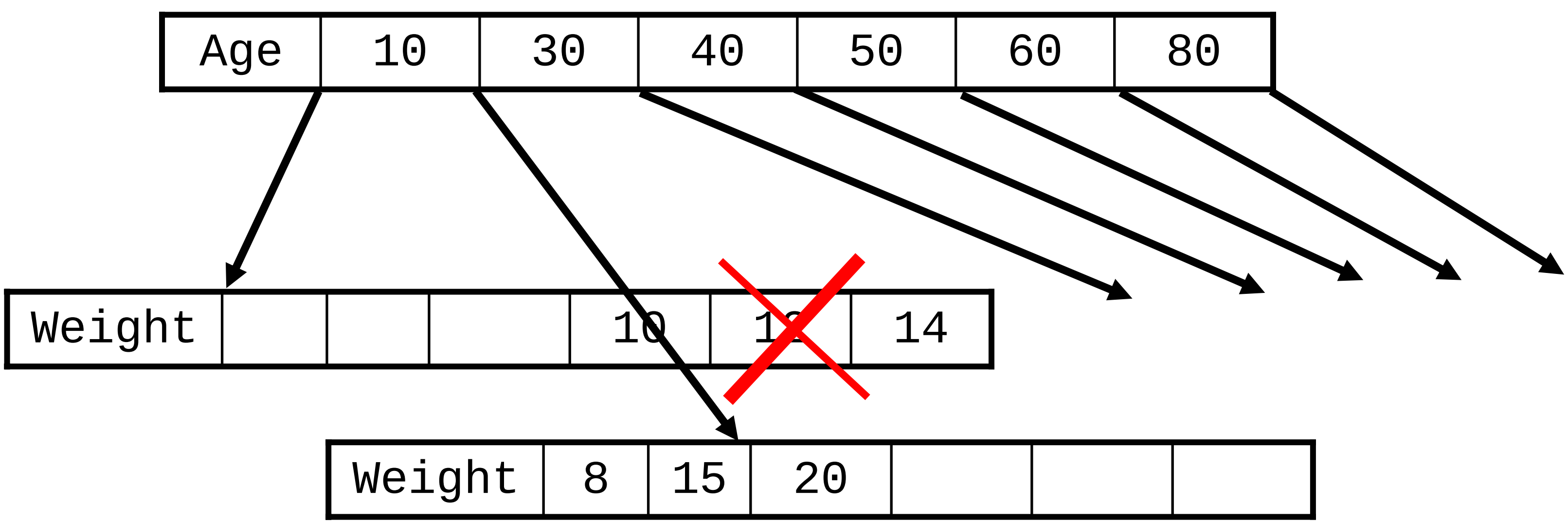
- Inserting a point
  - Search data block of leaf
  - If space available – done
  - Otherwise, chose split dimension and position for this block
    - This is a local decision, but remains stable for the future
    - Find dimension and split that divides set of points into two sets
      - Consider current points and split in two sets of approximately equal size
      - Consider known distributions of values in different dimensions
      - Use alternation scheme for dimensions
      - Finding “optimal” split points is expensive for high dimensional data (point set needs to be sorted in each dimension) – use heuristics
    - Wrong decisions in early splits lead to tree degradation
      - CS students at HU: Don’t split at sex, religion, place of birth, ...
    - But we don’t know which points will be inserted in future
      - Use knowledge on attribute value distributions

kd-Tree Deletions

- Deleting points
  - Search data block and delete point
  - If block becomes empty
    - Leave it – bad points/space ratio
    - Delete block and parent node
      - Changes height of tree – danger of tree degradation
    - Consider sibling in tree and reorganize
      - Touches more blocks
- Keeping kd-trees balanced is difficult
  - Usually, some degradation is accepted
- Improvements for kd-trees on secondary storage??
  - Option 2 is kdb tree – a balanced, IO-optimized kd tree

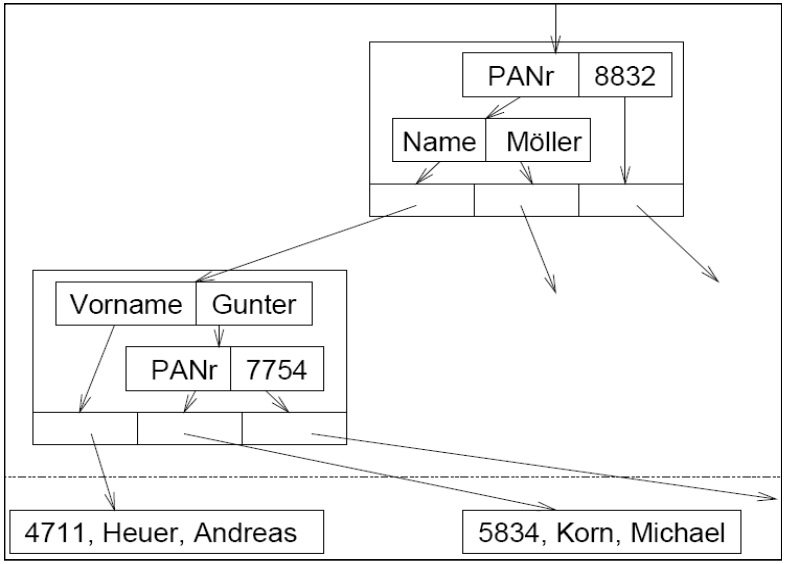
Fill Inner Blocks

- Option 1: Multiway branching
  - Split chosen dimension at r positions
    - r: Number of pointer/value pairs fitting in block
    - When sibling nodes need to be merged,
      - Split points of children usually are incompatible
      - Reorganization of subtrees required



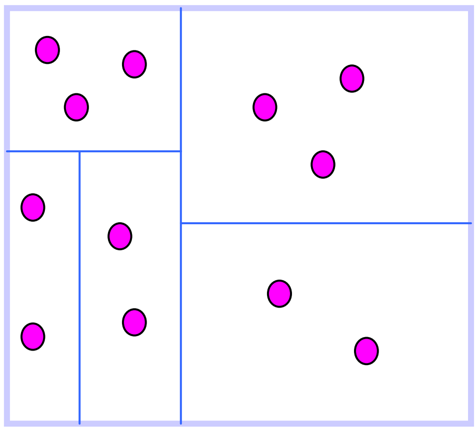
kdb trees

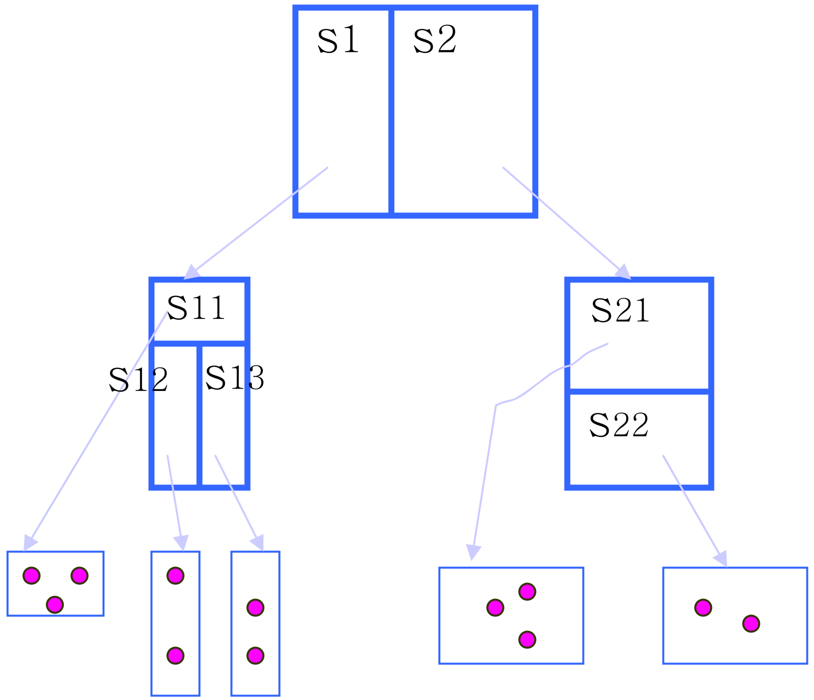
- Option 2: Store entire subtrees in one block
  - Inner nodes still have only two children
    - But those are (usually) stored in the same block
    - We need to “map” nodes to trees
  - kdb-tree: inner nodes store kd-trees
- Operations
  - Searching: As with kd trees
    - But on average better IO complexity
  - Insertion/Deletion
    - Complex schemes for keeping balance in tree (later)



Another View

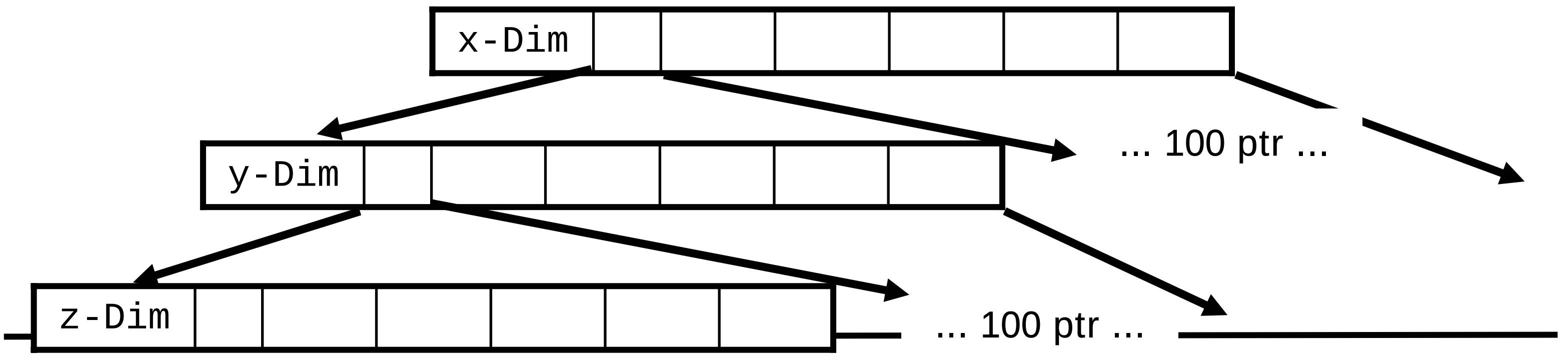
- Inner nodes define (possibly open) bounding boxes on subtrees
- kdb tree is a hierarchical index structure





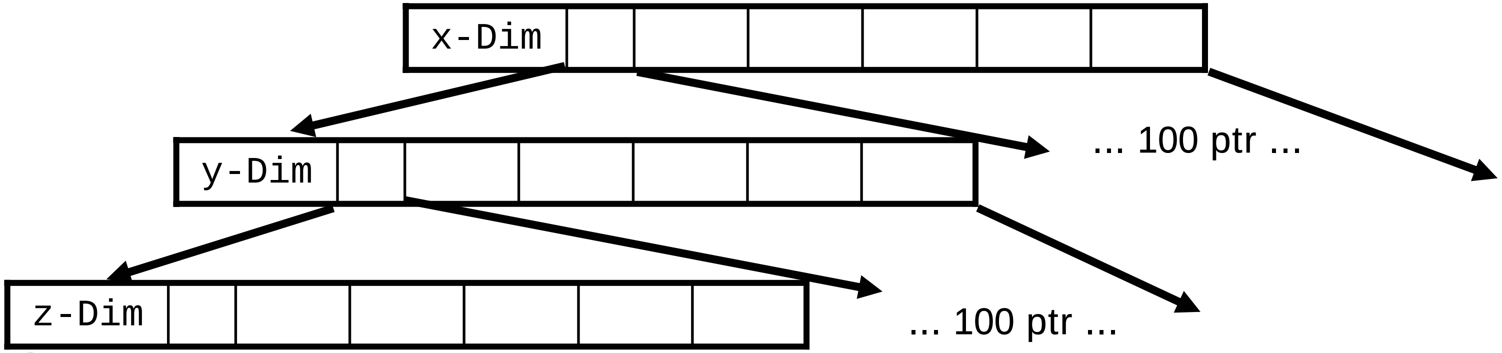
Example – Composite Index

- Consider 3 dimensions, n=1E7 points, block size 4096, |point|=9, |b-ptr|=10
  - We need ~22000 leaf blocks
- Composite B* index
  - Inner blocks store at least 100 pointers (max ~220)
  - We need 3 levels (2nd level has 10.000 pointers)
  - With uniform distribution, 1st level will mostly split on 1st dimension, 2nd level on 2nd dimension, 3rd level on 3rd dimension
- Box query in all three coordinates, 5% selectivity in each dimension
  - We read 5% of 2nd level blocks = 5 IO
  - For each, we read 5% of 3rd level blocks = 5*5=25 IO
  - For each, we read 5% of data blocks = 125 IO
  - Altogether: 155 IO



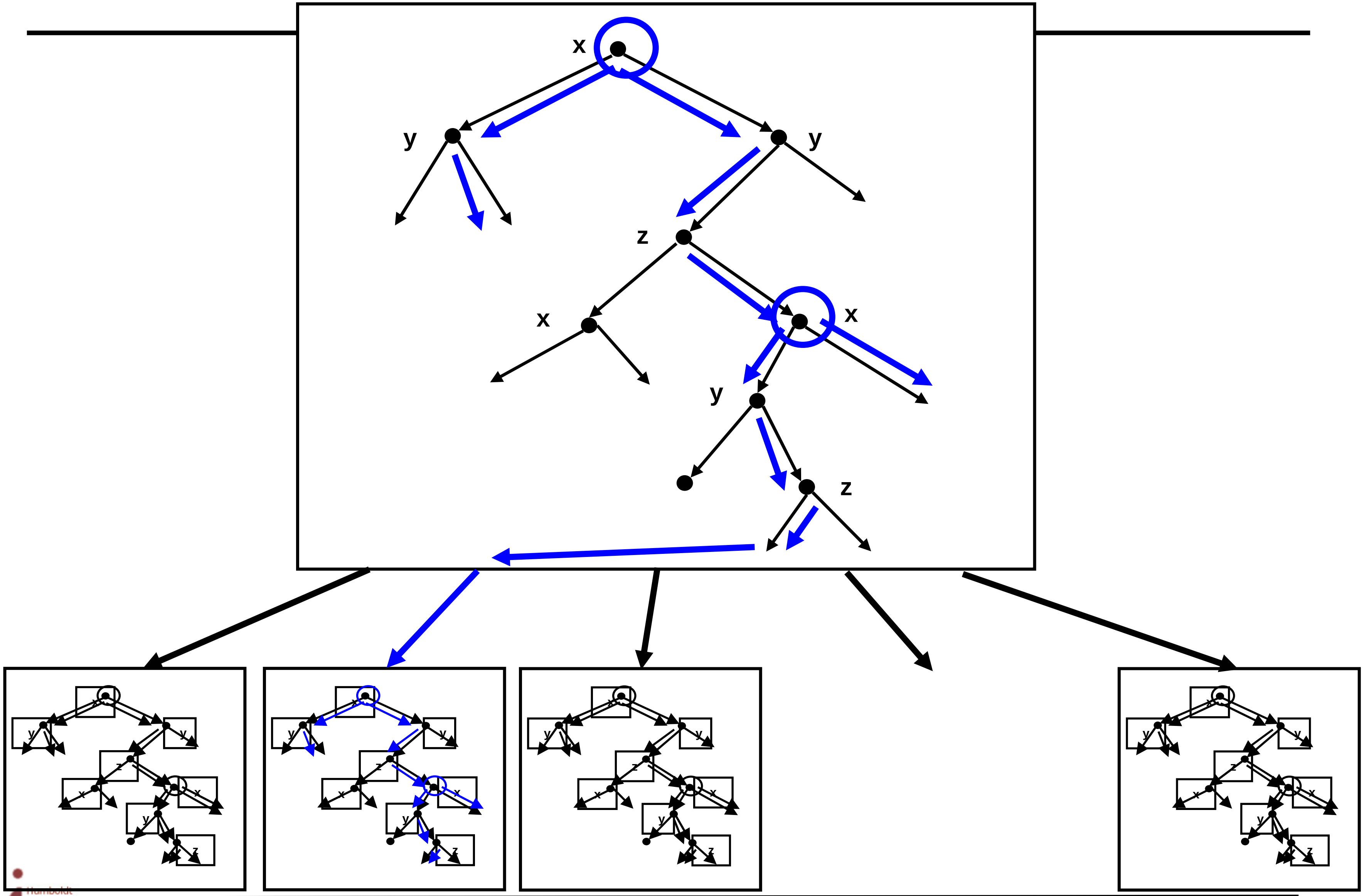
Example Partial Box Query

- Box query on 2nd and 3rd dimensions only, asking for a 5% range in each dimension
  - We need to scan all 100 2nd level blocks
    - Each 2nd level block contains the 5% range
  - Next, we scan 5% of 3rd level blocks = 500 blocks
    - We follow 5% of pointers from 2nd level blocks
  - For each, we read 5% of data blocks = 2500 data blocks
  - Altogether: 3100 IO
- Note: 5% selectivity in 2 dims means 0.0025 selectivity altogether = 25000 points
  - Only 60 blocks if optimally packed



Example – kdb-tree

- Balanced tree will have ~14 levels
  - ~400 points in one block (assume optimal packaging)
  - We need to address 1E7/400 = 25.000~2^{14} blocks
- Consider $128=2^7$ inner nodes in one block
  - Rough estimate; we need to store two b-ptr for each inner node, but most b-ptr are just offsets into the same block
- kdb tree structure
  - 1st level block holds 128 inner nodes = levels 1-7 of kd tree
  - There are 128 2nd level blocks holding levels 8-14 of kd tree
- 1st block evenly splits space in 128 regions
- Box query in all three coordinates, 5% selectivity in each dimension
  - Overall selectivity is $(0.05)^3$ = 0,000125% of all points (1250 points)
  - Very likely, we need to look at only ~1 2nd level block
    - On 7 levels, 2 dim. will have been split into 4 sections, one dim. into 8
    - If query intersects with split planes in each dimension: worst case 8 IO
    - Example: 100 values in one dimension, split will be at 25,50,75, query region covers 5 consecutive values – only 30 of 95 such regions cross a split point
  - Very likely, we need to look at only 4 data blocks (holding together the 1250 points)
  - Altogether: 1+1+4 = 6 IO (compared to 155 for composite index)


x
y
y
z
x
x
y
z

Example - Partial Box Query 2

- Box query on 2nd and 3rd dimensions only, asking for a 5% range in each dimension
  - In first block (7 levels), we will have ~2 splits in each dimension (2 times 2, 1 time 3)
    - Assume we miss the dimension with 3 splits
  - Hence, in ~4 of 7 splits we know where we need to go, in ~3 we need to follow both children
  - We need to check only $2^3=8$ second-level blocks
    - Again – number gets higher when query range crosses split plane
    - The less likely, the lower the selectivity is
  - Same argument holds in 2nd level blocks = 8*8 data blocks
  - Altogether: 1+8+64 = 73 IO
    - We almost reach optimum with 60 blocks
    - Compare to 3100 for composite index
- Beware
  - We made many, many assumptions
  - Layout of subtrees to nodes rarely optimal
  - Performance can greatly vary due to n# of dimensions, distributions, order of insertions and deletions, selectivity, split and merge policies, ...

Conclusion

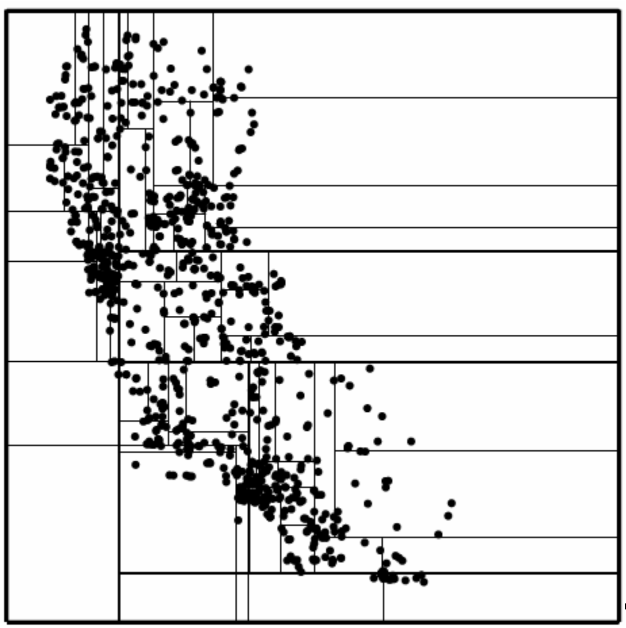
- Kdb trees can be perfectly balanced
  - Similar method as for b* trees
  - When splitting a leaf, a new node must be inserted into parent
  - Overflow may walk up to root
  - When inner nodes are split, splits must be propagated downward
    - As regions need to stay convex
- Kdb trees have problem with fill degree
  - Many insertions/deletions lead to almost empty leaves
  - Index grows unnecessary large
  - No guarantee for lowest fill degree as in b* tree

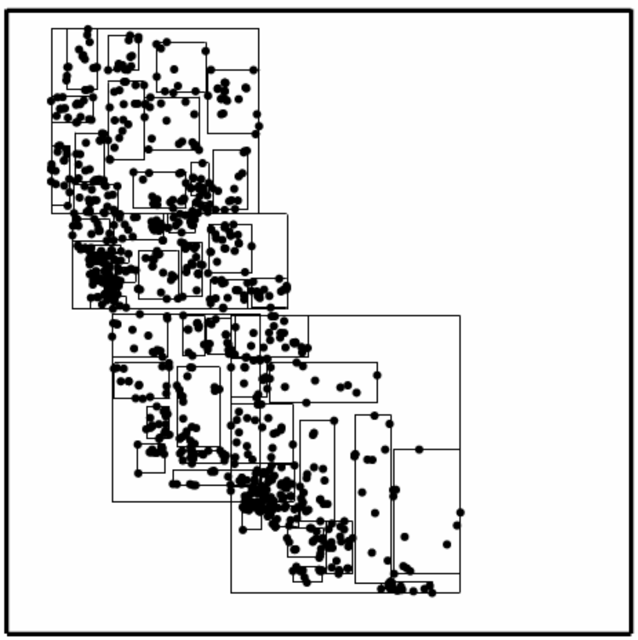
Content of this Lecture

- Introduction to multidimensional indexing
- Partitioned Hashing
- Grid files
- kd and kdb Trees
- R trees

R-Trees

- Can store geometric objects (with size) as well as points
- Each object is stored in exactly one region on each level
- Since sized objects may overlap, R tree regions may overlap
- Better adaptation to distribution of data objects
- Only those hyperregions containing data objects are represented
- Many variations (see literature)





General Idea

- R-trees store n-dimensional rectangles
  - For geometric objects, use minimal bounding box (MBB)
- Objects in a region of the n-dimensional space are stored in a block
  - The region borders is the MBB of all objects in contains
  - Regions may overlap – see below
- Regions are recursively contained in larger regions
  - Tree-like shape
  - Region borders in each level are MBB of all child regions
  - Regions are only as large as necessary
  - Regions of a level need not cover the space completely
- Regions in one level may overlap
  - Or not – variation of classical R tree
  - Without overlaps: much more complicated insertion/deletion, but better search complexity
- Finding all rectangles overlapping with a query rectangle
  - In each level, intersect with all regions
  - More than one region might have non-empty overlap
    - All must be considered
    - In general, no O(log(n)) complexity

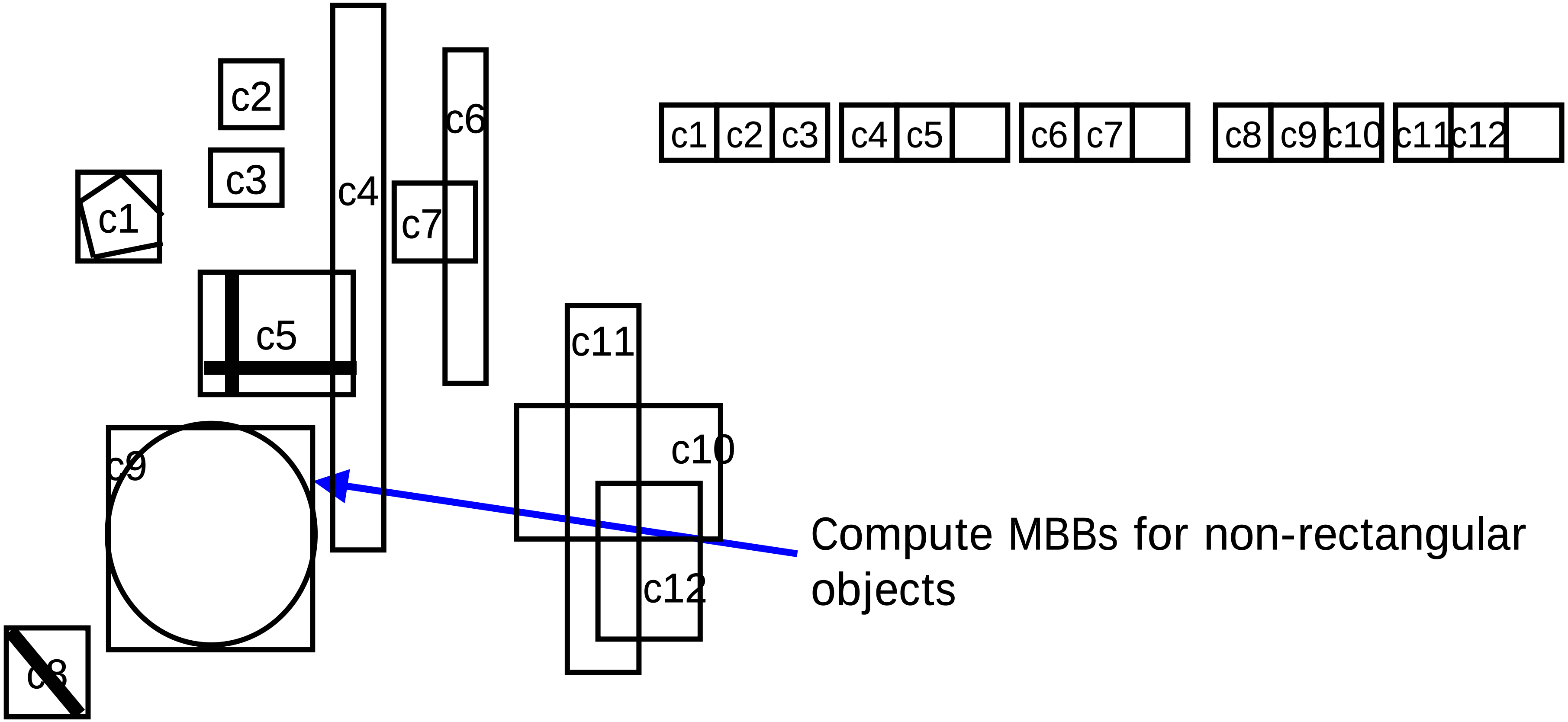
Inserting an Object

- In each level, find regions that contains object
  - Completely or partly
  - More than one region with complete overlap
    - Chose one (smallest?) and descend
  - None with complete, but several with partial overlap
    - Chose one (largest overlap?) and descend
  - No overlapping region at all
    - Chose one (closest?) and descend
  - We insert object in only one region
- In leaf node with space available
  - Insert object and adapt MBB
  - Recursively adapt MBBs up the tree
  - This generates larger and larger overlaps – search degrades
- In leaf node with no space available
  - Split block in two regions
  - Compute MBBs
  - Can affect MBB of higher regions – ascend recursively

Other Operations

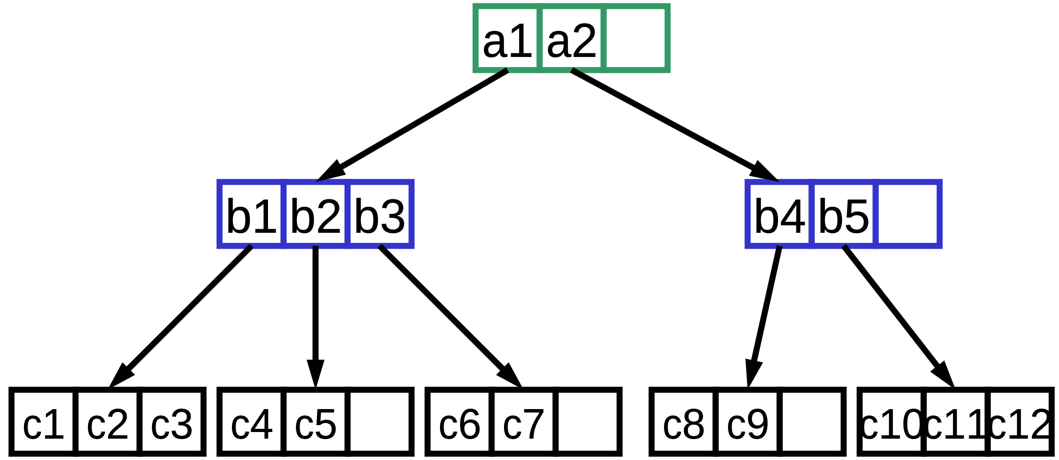
- Deleting an object
  - Equally complicated
- Balancing the R-tree
  - Much more complicated

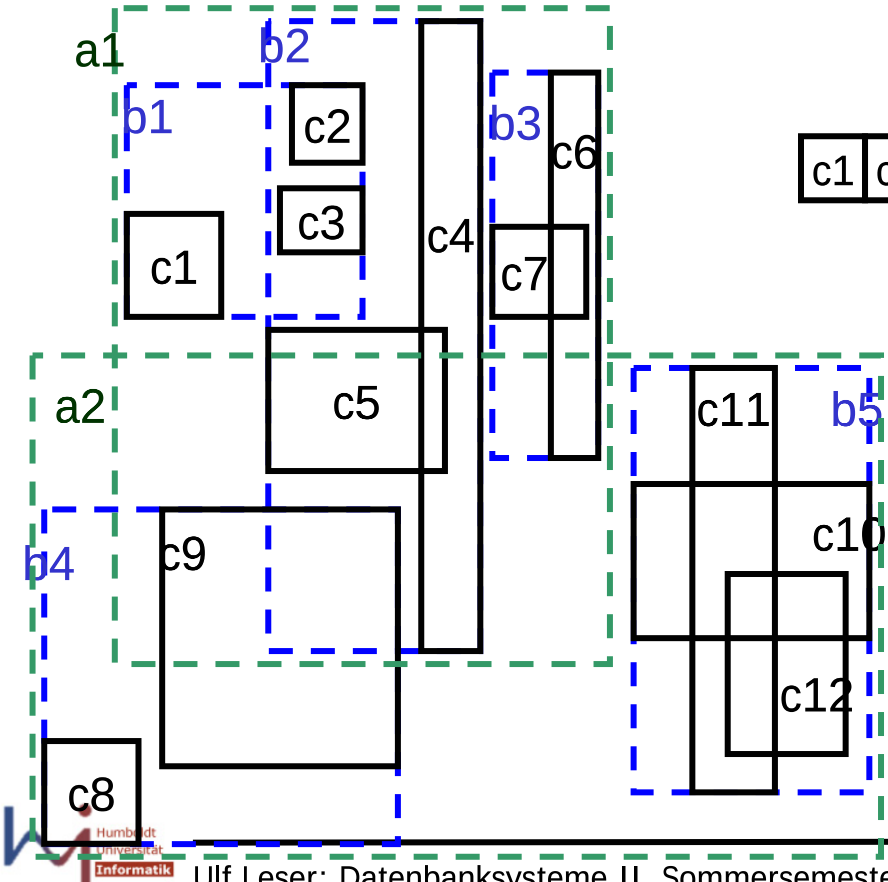
Example (from Donald Kossmann)



Compute MBBs for non-rectangular objects

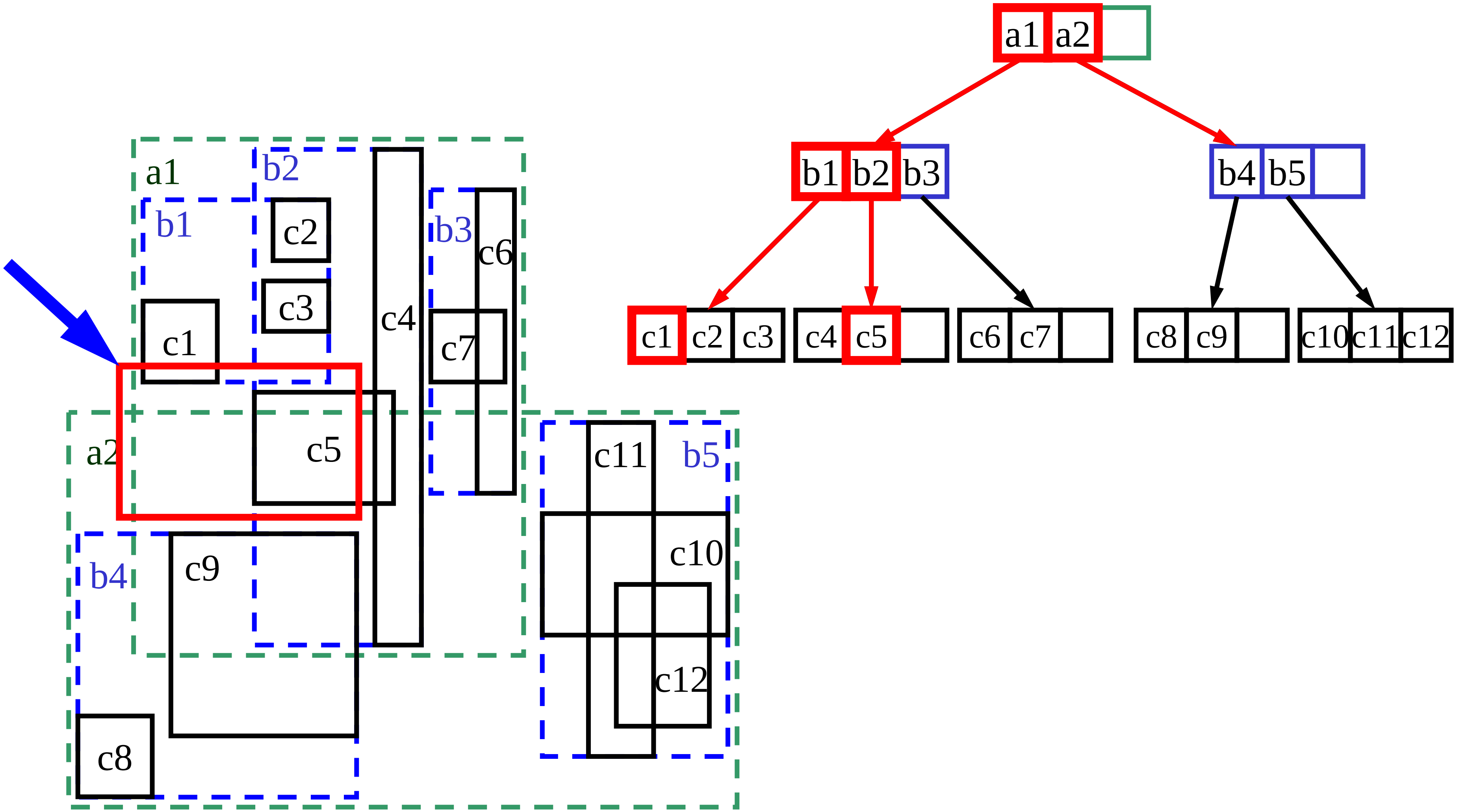
Example: Regions



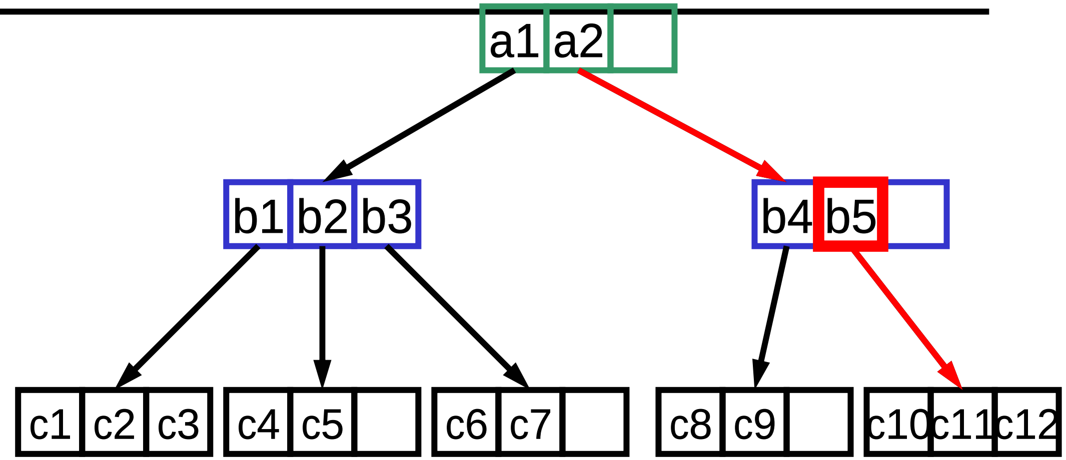


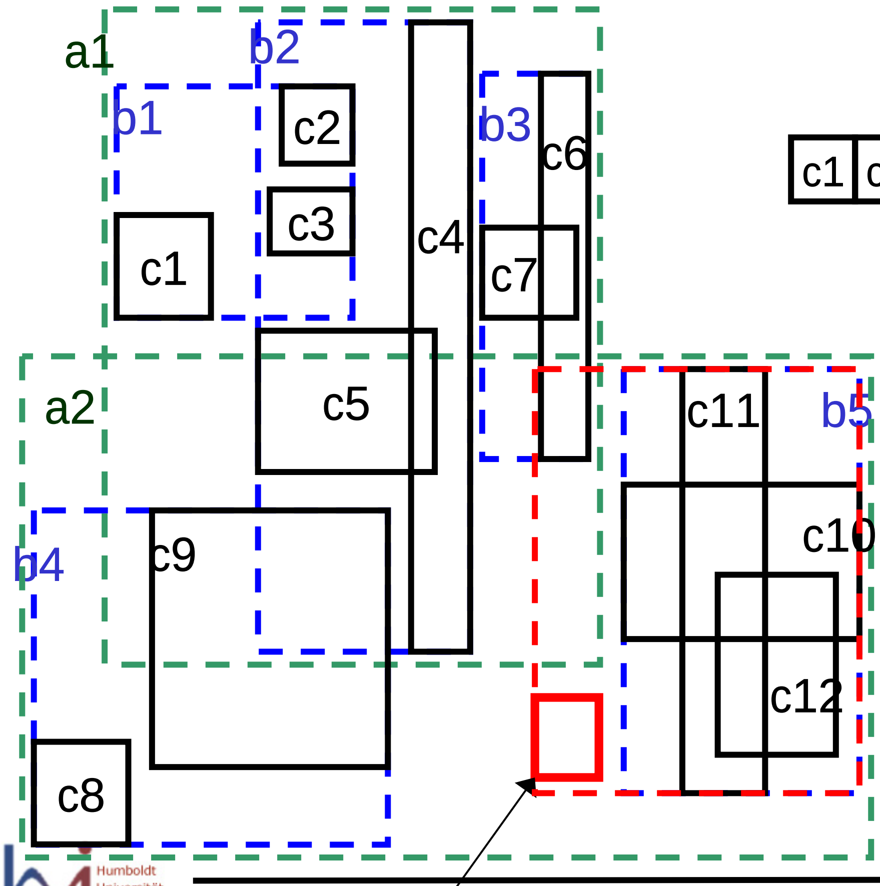
- Objects are hierarchically grouped into regions
- Regions may overlap
- Objects are represented only once

Example: Searching



Example: Insertion, Search Phase





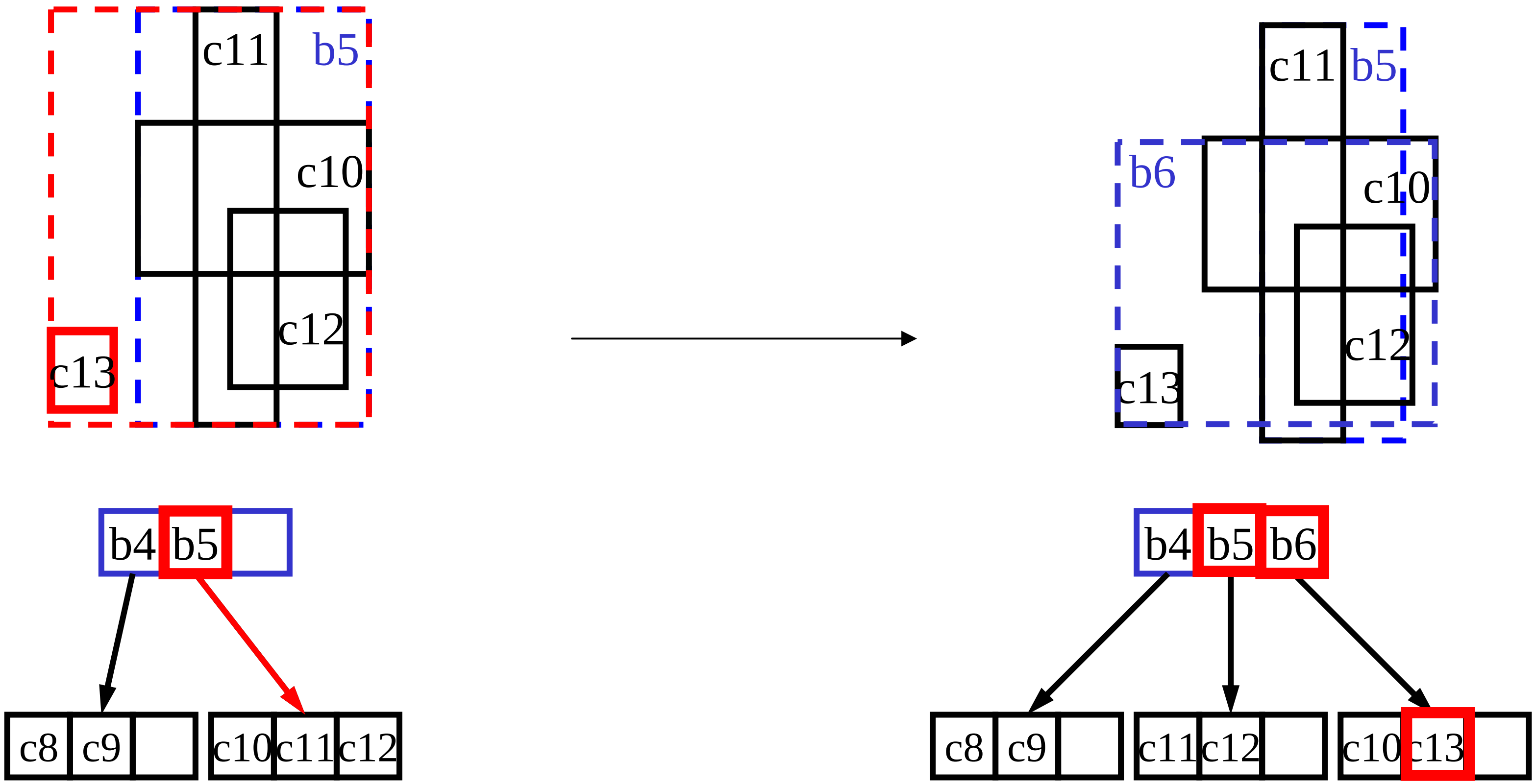
- Search regions whose MBB must be expanded the least
- Repeat on each level (do not adapt MBBs yet)

Overflow, split required

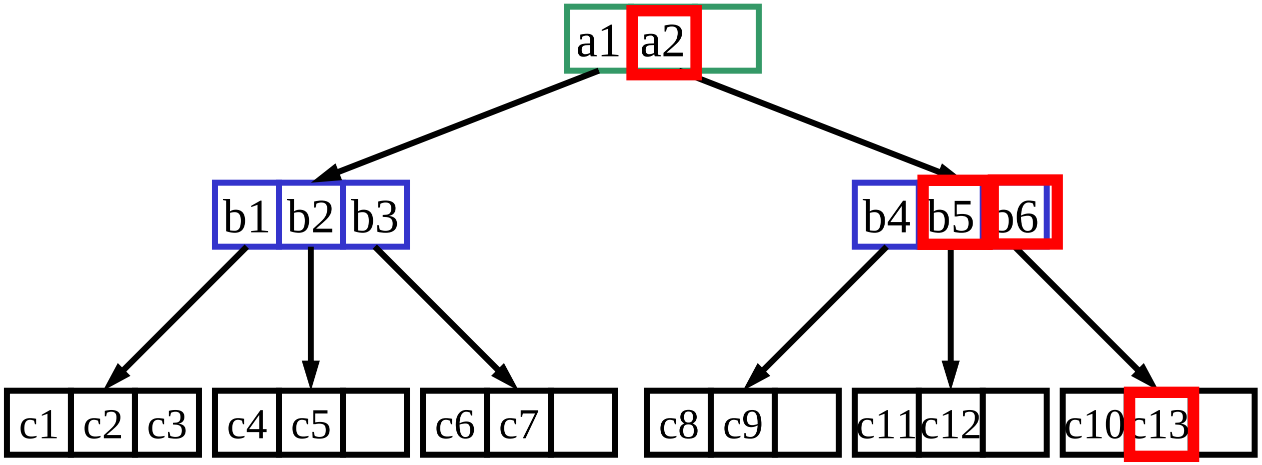
Note: Having chose b4 would avoid split – but how can we know?

Example: Insertion, Split Phase

Usually , several splits are possible



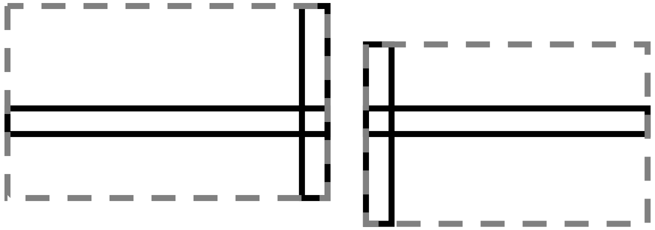
Example: Insertion, Adaptation Phase

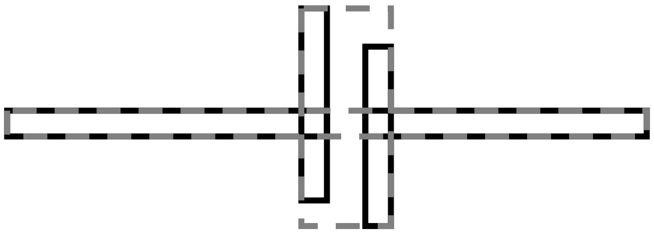


- MBBs of all parent nodes must be adapted
- Block split might induce node splits in higher levels of the tree

Block Splits

- Problem: How should we optimally split a overflow-node into two regions?
- Option 1: Avoid overlaps, cover large space
  - Compute partitioning such that there exists a separating hyperplane
  - Minimizes necessity to descend to different children during search
  - Generally requires larger regions – search in empty regions is detected later
- Option 2: Allow overlaps, minimize space coverage
  - Compute partitioning such that sum of volumes of MBBs is minimal
  - Overlaps increases changes to descend on multiple paths during search
  - But: Unsuccessful searches can stop early





Block Splits

- Whatever strategy we chose
  - Consider a block with n objects
  - There are 2^n possibilities to partition this block into two
  - Most strategies require to check them all
  - Use heuristics instead of optimal solution
- R* tree
  - Chose as criterion combination of sum of covered spaces, space of intersection, and sum of girt
  - Use heuristic for concrete decision
  - Currently best strategy (still?)

Multidimensional Data Structures Wrap-Up

- We only scratched the surface
- Partitioned Hashing, Gridfile, kdb-Tree, R-Tree
- Other: X tree, hb tree, R+ tree, UB tree, ...
  - Store objects more than once; other than rectangular shapes; map coordinates into integers; ...
- Curse of dimensionality
  - Your intuition plays tricks on you
  - The more dimensions, the more difficult
    - Balancing the tree, finding MBBs, split decisions, etc.
  - All structures begin to degenerate somehow
    - Exploding size of directories, linear kdb-trees, all regions overlap, ...
  - Often, linear scanning of objects is quicker
    - Or: Compute lower-dimensional, relationship-preserving approximations of objects and filter on those

Example

- Assumption: When deleting in object in R-tree, the new MBB will probably not be smaller, since most objects are far from the borders of the region
- Consider cubes and define border as within 10% of border
- In a 1-dimensional interval, 80% or points are not at the border
- In a 2-dimensional rectangle, 64%
- d=5: 32%; d=32: <0.001% - all points are at some border