Information Retrieval

Language Models

Ulf Leser

Content of this Lecture

- Language Models
- Markov Models
- Data sparsity
- Language Models for IR

- Most material from [MS99], Chapter 6

Problem

- Given a prefix of a sentence: Predict the next word
  - "At 5 o'clock, we usually drink …"
    - "tea" – quite likely
    - "beer" – quite unlikely
    - "a beer" – slightly more likely, but still
    - "biscuits" – semantically wrong
    - "the windows need cleaning" – syntactically wrong
- Similar to Shannon's Game: Given a series of characters, predict the next one (used in communication theory)
- Abstract formulation: Given a language L and the prefix S[1..n] of a sequence S, $S \in L$: Predict S[n+1]
- This is a ranking problem – no single solution

Applications

- Speech/character recognition
  - Given a transcribed prefix of a sentence – which word do we expect next?
- Automatic translation
  - Given a translated prefix of a sentence – what do we expect next?
- T9: "… information about common word combinations can also be learned …"
- General: Use probabilities of next word as a-priori probability for interpreting the next signal
  - Helps to disambiguate between different options
  - Helps to make useful suggestions
  - Helps to point to possible errors (observation ≠ expectation)

Language Models

- Classical approach: Grammars
  - Regular, context-free, …
  - Grammars can be learned from examples
    - Not trivial, underdetermined, not covered here
  - Usually, multiple continuations of a prefix are allowed
- (Deterministic) Grammars do not help in deciding which is the most probable one
  - Many equally probable continuations remain
- Better: Probabilistic grammars
  - Probabilistic automata: Transitions have a relative frequency
  - Hint: Markov models are probabilistic automata

N-Grams over Words

- Popular and simple approach: N-gram models
  - "Indeed, it is difficult to beat a trigram model on the purely linear task of predicting the next word" [MS99]
  - This statement is not true anymore today – large language models
- Definition
  A (word) n-gram is a sequence of n words.
- Usage
  - Count frequencies of all n-grams in a corpus of the language
    - Slide window of size n over text and keep counter for each n-gram ever seen
  - Given a sentence prefix, predict most probable continuation(s) based on n-gram frequencies – how?

N-Grams for Language Modeling

- Assume a sentence prefix with n-1 words <w_1,…,w_{n-1}>
- Look-up counts of all n-grams starting with <w_1,…,w_{n-1}>
  - I.e., n-grams <w_1,…,w_{n-1},w_n>
- Choose that w_n whose n-gram is the most frequent
- More formally
  - Compute, for every possibly w_n,

$$p(w_n) = p(w_n \mid w_1,\ldots,w_{n-1}) = \frac{p(w_1,\ldots,w_n)}{p(w_1,\ldots w_{n-1})}$$

  - Choose w_n which maximizes $p(w_n)$

Which n?

- In language modeling, one usually chooses n=3-4
- That seems small, but most language effects are local
  - But not all: “Dan swallowed the large, shiny, red …” (Car? Pil? Strawberry?)
- Also, we can hardly obtain robust relative counts for larger n - not enough training data
  - Data sparsity problem
  - Some remedies later
  - In high dimensional problems, training data is always sparse

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History and Applications

- Andrej Andrejewitsch Markov (1856-1922)
  - Russian Mathematician
  - Developed Markov Models (or Markov Chains) as a method for analyzing language
  - Markov, A. A. (1913). "Beispiel statistischer Untersuchungen des Textes ‚Eugen Onegin‵, das den Zusammenhang von Ereignissen in einer Kette veranschaulicht (Original in Russisch)." *Bulletin de l'Academie Imperiale des Sciences de St.-Petersbourg*: 153-162.
- Markov Models and Hidden Markov Models are popular in
  - Language Modeling, Part-of-speech tagging
  - Speech recognition
  - Named entity recognition / information extraction
  - Biological sequence analysis
  - Currently overcome by neural networks: RNN, LSTM, transformer

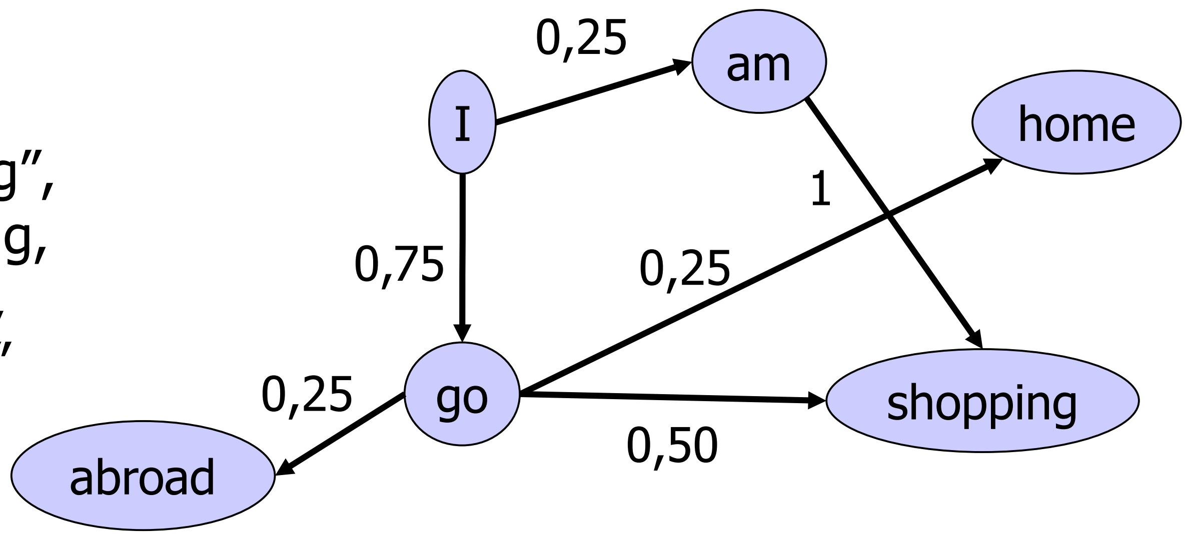
Markov Models

- Definition
  Assume an alphabet Σ. A Markov Model of order 1 is a sequential stochastic process with $|\Sigma|$ states $s_1, \ldots, s_n$ with
  - *Every state emits exactly one symbol from Σ*
  - *No two states emit the same symbol*
  - *For a sequence $<w_1,w_2,\ldots>$ of states, the following holds*

    $$p(w_n=s_n|w_{n-1}=s_{n-1}, w_{n-2}=s_{n-2},\ldots, w_1=s_1) = p(w_n=s_n|w_{n-1}=s_{n-1})$$
- Remarks
  - $a_{i,j} = p(w_n=s_j|w_{n-1}=s_i)$ are called transition probabilities
  - States and symbols have a 1:1 relationship
  - In language modeling, Σ = vocabulary = all words of a language

Visualization

- Since every state emits exactly one symbol (word) and vice versa, we merge states and words
- State transition graph
  - Nodes are states labeled with the word they emit
  - Edges are transitions labeled with a probability
  - We only draw edges with probability larger than 0
- Example
  - "I go home",
    "I go shopping",
    "I am shopping,
    "I go abroad",
    "Go shopping"



Probability of a Sequence of States (=a Sentence)

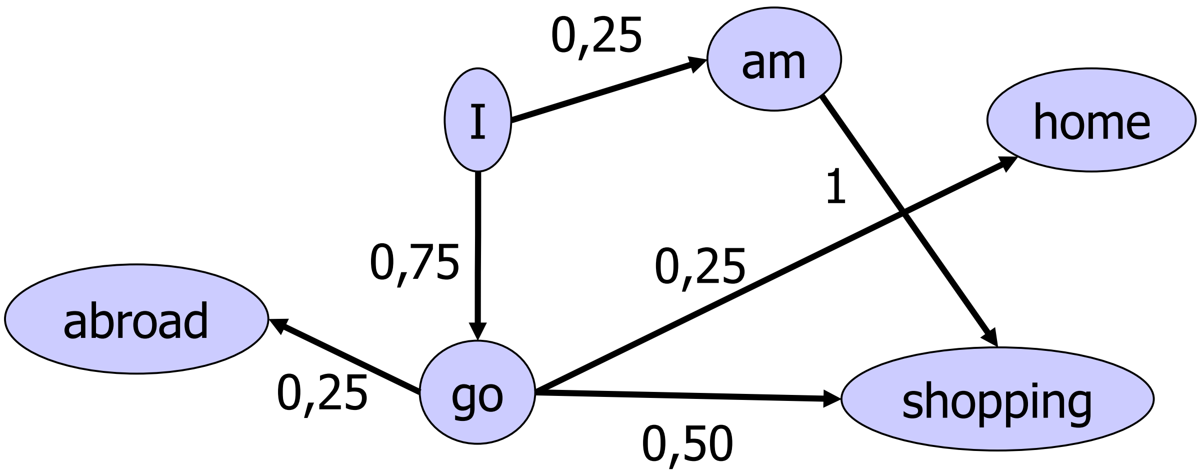
- Assume a Markov Model M of order 1 and a sequence S of states with |S|=n
- With which probability was S generated by M, i.e., what is the value of p(S|M)?

$$p(S \mid M) = p(w_1 = S[1]) * \prod_{i=2..n} p(w_i = S[i] \mid w_{i-1} = S[i-1])$$

$$= a_{0,S[1]} * \prod_{i=2..n} a_{S[i-1],S[i]} = a_{0,1} * \prod_{i=2..n} a_{i-1,i}$$

- Requires start probabilities $a_{0,i}$ for all words
  - Either assume some probability for all words
  - Or count relative frequency of words at beginning of sentences

Example



- Assume global start probability of 1
- p("I go home") = p(w_1=„I"|w_0)* p(w_2=„go"|w_1=„I") * p(w_3=„home"|w_2=„go")
  = 1 * 0.75* 0.25 = 0.1875
- Problem: Pairs we have not seen in training get prob. 0
  - Example: "I am abroad"
  - With such a small "corpus", too many transitions get p=0

Stochastic Processes

- Consider language generation as a sequential stochastic process
- At each stage, the process generates a new word
  - Like a DFA, but transitions have probabilities
- Question: How big is the memory? How many previous words does the process use to determine the next step?
  - 0: Markov chain of order 0: No memory at all
  - 1: Markov chain order 1: Next word only depends on prev. word
  - 2: Markov chain order 2: Next word only depends on 2 prev. words
  - …

Higher Order Markov Models

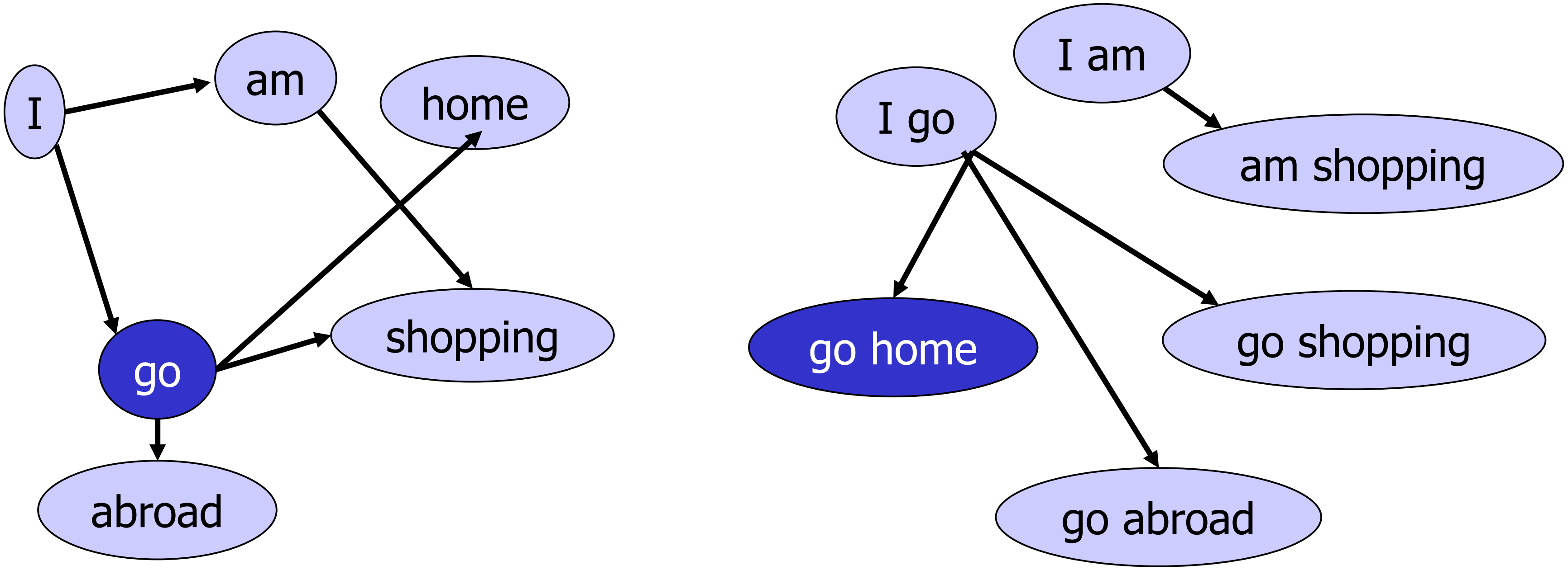
- Markov Models of order k, k>1
  - The probability of being in state s after n steps depends on the k predecessor states $s_{n-1},\ldots s_{n-k}$

    $p(w_n=s_n|w_{n-1}=s_{n-1}, w_{n-2}=s_{n-2},\ldots, w_1=s_1) = p(w_n=s_n|w_{n-1}=s_{n-1}, \ldots, w_{n-k}=s_{n-k})$
- We can easily transform any order k model M (k>1) into a Markov Model of order 1 (M')
  - M' has $O(|M|^k)$ states (all combinations of states of length k)
  - When learning transition probabilities, form a corpus, not all states will be reachable
  - Many are grammatically forbidden / semantically weird
    - "Der Mann Mann geht geht trinken grün"
    - "Der blaue Wein ratter lila"

Predicting the Next State

"I go home",
"I go shopping",
"I am shopping,
"I go abroad",
"Go shopping"

- The problem of language modeling is a bit different
- We do not want to reason about an entire sequence, but only about the next state, given a prefix of previous states
- N-gram model = Markov Model order n



Problem

- We learn our transition probabilities from a limited sample
- Thus, we only estimate the true transition probabilities
- Introduces an estimation error which we should reduce
  - Problem is researched a lot in statistics
- Extreme: Transitions we do not see at all in the corpus
  - Get a probability of 0
  - Will never be predicted
  - This does not mean that they are non-existing in the language

Importance of Data Sparsity

- How many n-grams do exist in principle?
  - Assume a language of 20.000 words
  - n=1: 20.000, n=2: 4E8, n=3: 8E12, n=4: 1.6E17, …
  - Rough numbers: Natural languages have many more words, but most combinations are not allowed
- In natural language corpora, almost all n-grams with n>4 are very sparse
  - Exponential growth cannot be balanced by "use larger corpora"
  - Especially n-grams over rare words are prone to be overlooked
- Trade-off
  - Large n: More expressive model, but bad transition estimations
  - Small n: Less expressive model, but better transition estimations

Example

- Unigrams: Always the most frequent word in the corpus, does not differentiate

In person	*she*		*was*		*inferior*		*to*		*both*		*sisters*	
1-gram	P(·)		P(·)		P(·)		P(·)		P(·)		P(·)	
1	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034
2	to	0.032	to	0.032	to	0.032	to	0.032	to	0.032	to	0.032
3	and	0.030	and	0.030	and	0.030			and	0.030	and	0.030
4	of	0.029	of	0.029	of	0.029			of	0.029	of	0.029
…												
8	was	0.015	**was**	**0.015**	was	0.015			was	0.015	was	0.015
…												
13	**she**	**0.011**			she	0.011			she	0.011	she	0.011
…												
254					both	0.0005			**both**	**0.0005**	both	0.0005
…												
435					sisters	0.0003					**sisters**	**0.0003**
…												
1701					**inferior**	**0.00005**						
2-gram	P(·|*person*)		P(·|*she*)		P(·|*was*)		P(·|*inferior*)		P(·|*to*)		P(·|*both*)	
1	and	0.099	had	0.141	not	0.065	**to**	**0.212**	be	0.111	of	0.066
2	who	0.099	**was**	**0.122**	a	0.052			the	0.057	to	0.041
3	to	0.076			the	0.033			her	0.048	in	0.038
4	in	0.045			to	0.031			have	0.027	and	0.025
…												
23	**she**	**0.009**							Mrs	0.006	she	0.009
…												
41									what	0.004	**sisters**	**0.006**
…												
293									**both**	**0.0004**		
…												
∞					**inferior**	**0**						
3-gram	P(·|*In,person*)		P(·|*person,she*)		P(·|*she,was*)		P(·|*was,inf.*)		P(·|*inferior,to*)		P(·|*to,both*)	
1	Unseen		did	0.5	not	0.057	Unseen		the	0.286	to	0.222
2			**was**	**0.5**	very	0.038			Maria	0.143	Chapter	0.111
3					in	0.030			cherries	0.143	Hour	0.111
4					to	0.026			her	0.143	Twice	0.111
…												
∞					**inferior**	**0**			**both**	**0**	**sisters**	**0**
4-gram	P(·|*u,I,p*)		P(·|*I,p,s*)		P(·|*p,s,w*)		P(·|*s,w,I*)		P(·|*w,I,t*)		P(·|*I,t,b*)	
1	Unseen		Unseen		in	1.0	Unseen		Unseen		Unseen	
…												
∞					**inferior**	**0**						

Table 6.3 Probabilities of each successive word for a clause from *Persuasion*. The probability distribution for the following word is calculated by Maximum Likelihood Estimate *n*-gram models for various values of *n*. The predicted likelihood rank of different words is shown in the first column. The actual next word is shown at the top of the table in italics, and in the table in bold.

Example

- Bi-grams: Correct words often rank high, but not always

In person	*she*		*was*		*inferior*		*to*		*both*		*sisters*	
1-gram	P(·)		P(·)		P(·)		P(·)		P(·)		P(·)	
1	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034
2	to	0.032	to	0.032	to	0.032	**to**	**0.032**	to	0.032	to	0.032
3	and	0.030	and	0.030	and	0.030			and	0.030	and	0.030
4	of	0.029	of	0.029	of	0.029			of	0.029	of	0.029
...												
8	was	0.015	**was**	**0.015**	was	0.015			was	0.015	was	0.015
...												
13	**she**	**0.011**			she	0.011			she	0.011	she	0.011
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254					both	0.0005			**both**	**0.0005**	both	0.0005
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2-gram	P(·|*person*)		P(·|*she*)		P(·|*was*)		P(·|*inferior*)		P(·|*to*)		P(·|*both*)	
1	and	0.09[illegible]	had	0.141	not	0.065	**to**	**0.212**	be	0.111	of	0.066
2	who	0.099	**was**	**0.122**	a	0.052			the	0.057	to	0.041
3	to	0.076			the	0.033			her	0.048	in	0.038
[illegible]	in	0.04[illegible]			to	0.031			have	0.027	and	0.025
...												
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1	UNSEEN		UNSEEN		in	1.0	UNSEEN		UNSEEN		UNSEEN	
...												
∞					**inferior**	**0**						

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Example

In person	*she*		*was*		*inferior*		*to*		*both*		*sisters*	
1-gram	$P(\cdot)$		$P(\cdot)$		$P(\cdot)$		$P(\cdot)$		$P(\cdot)$		$P(\cdot)$	
1	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034
2	to	0.032	to	0.032	to	0.032	**to**	**0.032**	to	0.032	to	0.032
3	and	0.030	and	0.030	and	0.030			and	0.030	and	0.030
4	of	0.029	of	0.029	of	0.029			of	0.029	of	0.029
...												
8	was	0.015	**was**	**0.015**	was	0.015			was	0.015	was	0.015
...												
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4-gram	$P(\cdot\|u,I,p)$		$P(\cdot\|I,p,s)$		$P(\cdot\|p,s,w)$		$P(\cdot\|s,w,I)$		$P(\cdot\|w,I,t)$		$P(\cdot\|I,t,b)$	
1	UNSEEN		UNSEEN		in	1.0	UNSEEN		UNSEEN		UNSEEN	
...												
∞					**inferior**	**0**						

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- Tri-grams: Has a hit, but already suffers from sparsity
- Four-grams: Unusable

- Corpus: Fraction of Jane Austen's oeuvre, ~600.000 tokens, data from [MS99]

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Solutions we will not Discuss in Detail

- Reduce the number of words using stemming
  - Might help to go from n=3..4 to n=4…5
  - Important grammatical clues are lost
- Use some form of "binning" of tokens into classes and compute n-grams over token classes, not token
  - All numbers -> one class
  - All verbs -> one class (POS tags)
  - All verbs related to "movement" -> one class
  - Every synset from WordNet -> one class
  - Difficult …

Statistical Estimators

- We were a bit sloppy so far
- We want

$$p(w_n) = p(w_n \mid w_1,\ldots,w_{n-1}) = \frac{p(w_1,\ldots,w_n)}{p(w_1,\ldots w_{n-1})}$$

- But we only have $count(w_1,\ldots,w_n)$

- So far, we always implicitly assumed

$$p(w_1,\ldots,w_n) = \frac{count(w_1,\ldots,w_n)}{N}$$

  – N: all observed n-grams

MLE for N-gram Models

- This is called a Maximum Likelihood Estimator (MLE)
- MLE gives maximum likelihood to the training data
  - Gives zero probability to all events not in the training data
  - The probability mass is spent entirely on the training data
  - Overfitting
- Need to smooth the estimates to account for the limitations of the sample

Smoothing I: Laplace‘s Law

- Give some probability mass to unseen events
- Very simple suggestion: “Adding one”

$$p_{LAP}(w_1,\dots,w_n) = \frac{count(w_1,\dots,w_n)+1}{N+B}$$

  - Where B is the number of possible n-grams, i.e., K^n
- Result: All n-grams get a probability≠0
- But – moves too much mass to the unknown
  - Estimates for seen n-grams are scaled down dramatically
  - Estimates for unseen n-grams are small, but there are so many
    - And many of them are truly impossible
  - In a corpus of 40 M words with K~400T, 99.7% of the total probability mass is spend in unseen events

Smoothing II: Lidstone's Law

- Laplace not suitable if there are many events, but few seen
- Lidstone's law gives less probability mass to unseen events

$$p_{LIP}(w_1,\ldots,w_n) = \frac{count(w_1,\ldots,w_n)+\lambda}{N+\lambda * B}$$

  - Small λ: More mass is given to seen events
  - Typical estimate is λ=0.5
  - Appropriate values can be learned (next slide)
- Still: Estimate of seen events is linear in the MLE estimate
  - Not a good approximation of empirical distributions
- Other: Good-Turing Estimator, n-gram interpolations, …

Learning Appropriate Values for λ

- We "simulate" seen and unseen events
- Divide corpus in two disjoint parts C_1 and C_2
- Count frequencies of n-grams in C_1
- Let c be the number of n-grams from C_1 not present in C_2
- Set $\lambda=c/B$
  - The probability of an n-gram (in C_2) to be considered as not existing although in reality it does exist

Option III: Back-Off Models

- If we cannot find a n-gram with count≠0, use a (n-1)-gram
  - Or an n-2 gram, …
- Thus, in case there is no $p(w_1,…,w_n)≠0$, we "back off" to a simpler model

$$p(w_n \mid w_1,\ldots,w_{n-1}) = \frac{p(w_1,\ldots,w_n)}{p(w_1,\ldots w_{n-1})} \; or \; \frac{p(w_2,\ldots,w_n)}{p(w_2,\ldots w_{n-1})} \; or \; \frac{p(w_3,\ldots,w_n)}{p(w_3,\ldots w_{n-1})} \; or \ldots$$

  - Stop at the first (n-k)-gram with non-zero count
- Alternative: Always look at different n's
  - With different weights

$$p(w_n) = \lambda_1 \frac{p(w_{n-2}, w_{n-1}, w_n)}{p(w_{n-2}, w_{n-1})} + \lambda_2 \frac{p(w_{n-1}, w_n)}{p(w_{n-1})} + \lambda_3 p(w_n)$$

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??? AUSBAUEN

- Der Teil hier ist noch komisch. Es wird überhaupt nicht klar, wie man ein "richtiges" Sprchmodell da ienbauen könnte. Ich müsste also kurz Word Embeddings erwähnen – ich habe das ja vorher schon mal bei den Relevanzmodellen kurz erwähnt, hier aufgreifen
- Auch nochmal klar Machen – warum ist dsa hier anders als das probabilische Modell? Und wie nahe ist es am VSM? Dazu sollte es eine eigene Folie geben

New IR Model

- Recent trend in IR: Relevance based on language models
- Idea: See a document as a "language"
  - Learn a model M_d of this "language" (document)
  - Compute with which probability p(d|q) a given query has generated the model (=document)
  - Rank documents based on these probabilities
- Sounds weird, but leads to a simple, well justified, and powerful approach
  - Given the recent dramatically improved models learned over billion-word corpora
  - Very successful in recent evaluations
- Smoothing remains crucial (out-of-vocabulary error)

Approach

- If docs are small, only unigram models are sensible
- Model of a doc: Relative frequencies of all its words
- Compute

$$p(d \mid q) = \frac{p(q \mid d) * p(d)}{p(q)} \sim p(q \mid d) * p(d) \sim p(q \mid d)$$

  - p(q) is equal for all d – irrelevant for ranking
  - p(d) can be used to incorporate a-prior knowledge (e.g. prestige), but often is set to uniform – irrelevant for ranking
- We replace d with its model and obtain

$$p(q \mid d) = p(q \mid M_d) = p(k_1, k_2, \ldots, k_n \mid M_d) = \prod_{k \in q} p(k \mid M_d) = \prod_{k \in q} \frac{tf_{k,d}}{|d|}$$

Discussion

- Very simple
- Principled approach to justify usage of tf values
- More powerful for longer queries
- Problems
  - Words in q not in d: Smoothing
  - Where is idf gone?

Smoothing a Language Model for IR

- For instance, if $k \notin d$, set $p(k|M_d) = df_k/|D| = p(k|M_D)$
  - Token that are in d are counted with tf values (and not discounted with idf); tokens not in d are counted with df values
- More tunable parameters: Linear interpolation

$$p'(k \mid M_d) = \lambda * p(k \mid M_d) + (1-\lambda) * p(k \mid M_D)$$

  - Combine relevance of k in document and relevance of k in corpus
  - Large λ: More weight to the document, less weight to background
  - λ may vary, for instance with query size
- We are back at something similar to TF*IDF, but with a probabilistic interpretation, not a geometric one

Self Assessment

- What is language modelling about?
- Define a Markov model
- How can you turn a Markov model of order 4 into one of order 1?
- What is the data sparsity problem (in language modeling)?
- What is the disadvantage of Laplace smoothing?
- Explain how we can use language models for information retrieval