Algorithms and Data Structures

Data Types

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Content of this Lecture

- Example
- Abstract Data Types
- Lists, Stacks, and Queues
- Realization in Java

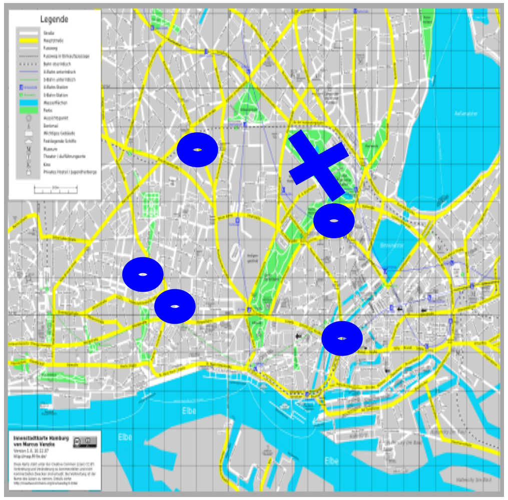
Problem

- Suppose you are in the centre of Hamburg and are looking for the next (i.e., closest) laptop repair shop
- Fortunately, your mobile knows your position and has a list of laptop repair shops in Hamburg
- How does your mobile find the closest shop?

Classical Post Box Problem

- Suppose a city with n boxes located at arbitrary positions
- You wake up in the middle of the city with a letter in your hand; the letter should be thrown in the closest post box
- How do you find the closest post box?
  - You have a list with locations of all post boxes
- Looking at a map is not the answer
- Devise an algorithm

```
S: set_of_coordinates;
c: coordinate (x,y)
…
```



Simple Solution

- How much work?

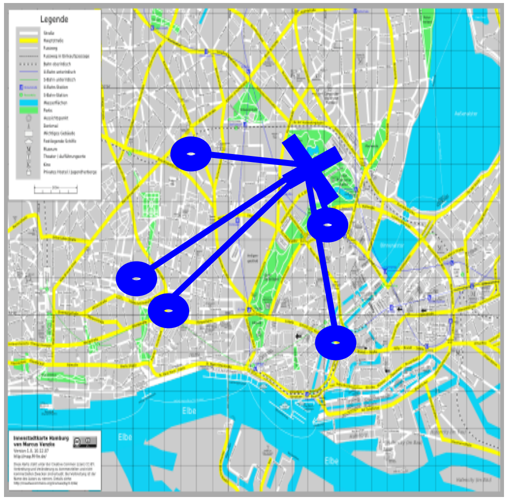
```
Input
  S: set_of_coordinates;
  c: coordinate (x,y);    # your loc
t: coordinate;            # closest box
m: real := MAXREAL;       # smal. dist
for each c'∈S do
  if m > distance(c,c') then
    m := distance(c,c');
    t := c';
  end if;
end for;
return t;
```

Simple Solution

```
Input
  S: set_of_coordinates;
  c: coordinate (x,y);    # your loc
t: coordinate;            # closest box
m: real := MAXREAL;       # smal. dist
for each c'∈S do
  if m > distance(c,c') then
    m := distance(c,c');
    t := c';
  end if;
end for;
return t;
```

- Clearly, we can save the second call to “distance”
- Thus, we need to compute |S| distances, make |S| comparisons, and perform at most 2*|S| assignments
- We perform O(|S|) operations, which are O(1) or distance computations

Simple Solution



- How much work?
- Clearly, we can save the second call to “distance”
- Thus, we need to compute |S| distances, make |S| comparisons, and perform at most 2*|S| assignments
- Euclidian distance
  - 6 arithmetic ops per distance

$$dist((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Not the only Option

- How much work?
- Clearly, we can save the second call to “distance”
- Thus, we need to compute |S| distances, make |S| comparisons, and perform at most 2*|S| assignments
- Manhattan distance
  - 5 operations, and different ones

$$dist((x_1, y_1),(x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2|$$

Complexity

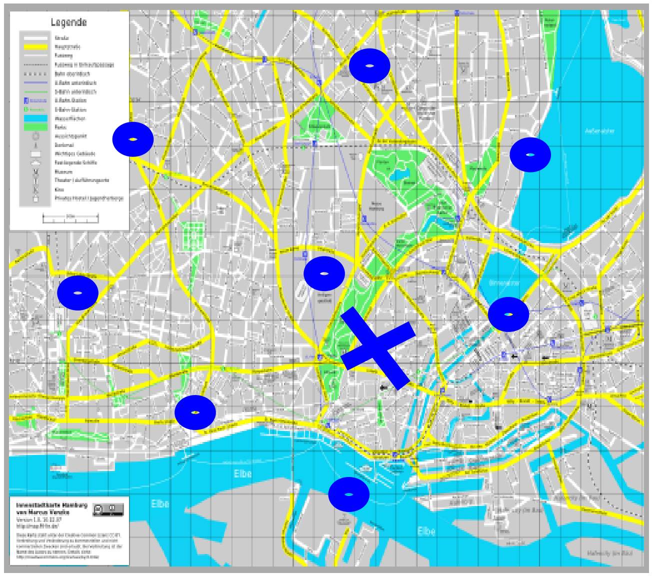
- How much work?
- Clearly, we can save the second call to “distance”
- Thus, we need to compute |S| distances, make |S| comparisons, and perform at most 2*|S| assignments
- Both cases: O(|S|*dim(S))
  - dim(S): Number of dimensions of every point in S

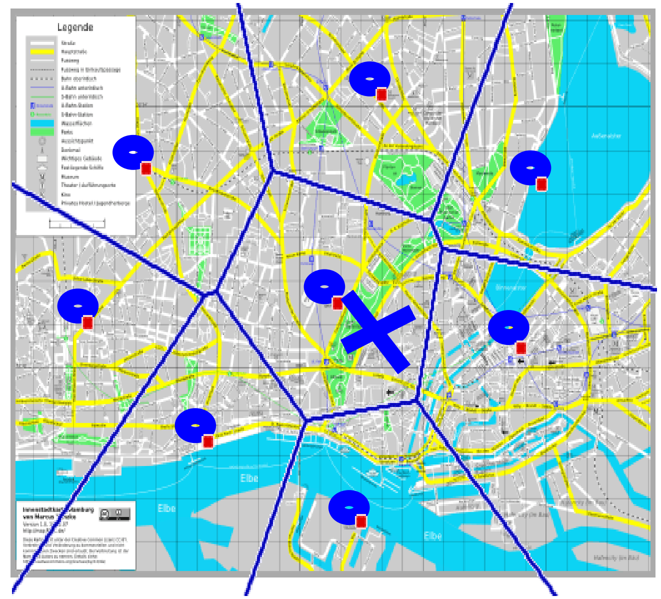
Data Structure Point of View

```
input
  S: set_of_coordinates;
  c: coordinate (x,y);
t: coordinate;
m: real := MAXREAL;
For each c'∈S do
  if m > dist(c,c') then
    m := dist(c,c');
    t := c';
  end if;
end for;
return t;
```

- Data structures
  - We need a list of coordinates
  - The algorithm must iterate over the elements of this list
  - A linked list would suffice
- Now assume we need to perform such searches very often
  - Can we represent S in another way (S'), such that searching requires less work?
  - Note: Time for computing S' from S will be ignored
    - Performed before searching starts
    - Assuming that S does not change

Voronoi Diagrams





- Pre-processing: Compute for every point $s \in S$ its Voronoi area, i.e., the area in which all points have s as nearest point from S
- Can be achieved in O(|S|*log(|S|)) time (no details here)
- Nearest-neighbor search using Voronoi diagrams is O(log(|S|))
- Conclusion: Finding a proper data structure does pay off

More Abstract

- We want a piece of software T that
  - can store a list of coordinates
  - can compute the nearest point to a given point c
  - Piece of software: (1) algorithm and (2) data structure
- Thus, T must support (at least) two operations
  - T.init (S: list_of_coordinates)
  - T.nearestNeighbor(c: coordinate): coordinate
  - T apparently uses another data structure: “coordinate”
- Such combinations of object sets and operations on these sets are called a data type
- If only look at sets and operations: Abstract data type
  - No details on algorithms / implementation

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Abstract Data Types (ADT)

- An ADT defines a set of operations over a set of objects of a certain (more basic) type
  - Or over multiple sets of objects of different or same types
- The set of operations and types is called signature
- An ADT is independent of an implementation
  - Different data structures to represent the objects
  - Different algorithms to implement the operations
  - An ADT is independent of any programming language
- Encapsulation: Objects are accessed only through the ops
- An implementation of a ADT is called a concrete (or physical) data type

Example

```
type points
import
  coordinate;
operators
  add:      points x coordinate → points;
  neighbor: points x coordinate → coordinate;
```

- ADT that we could use for our app for searching shops
- We only need two operations
  - A way to insert shops (with their coordinates)
  - A way to get the nearest shop with respect to a given coordinate
- We assume basic data types to be given (string, int ...)
- Not the only way ...

Modeling More Details

```
type shop
import
  coordinate;
  string;
operators
  getName: shop → string;
  getCoor: shop → coordinate;
```

```
type shops
import
  shop;
operators
  add:       shops x shop → shops;
  neighborC: shops x coordinate → coordinate;
  neighborN: shops x coordinate → string;
  neighborS: shops x coordinate → shop;
```

- An ADT defines what is necessary
- Designing a ADT is a modeling process
  - Shop owner? Laptop models being repaired? Opening hours?
  - Depends on requirements, ease-of-use, extensibility, personal preferences, existing ADTs, ...

Reusing Existing ADTs

- For implementing points (or shops), it would be helpful to import something that can hold a set of coordinates
- We need a list – an ADT in itself
  - A parameterized ADT– a list of elements of an arbitrary ADT T
  - For our ADT **`points`**, T will use objects of type **`coordinate`**

```
type list( T)
import
  integer, bool;
operators
  isEmpty:  list → bool;
  add:      list x T → list;
  delete:   list x T → list;
  contains: list x T → bool;
  length:   list → integer;
```

Axioms: What we Know about an ADT

- We expect operations on lists to have a certain semantic
  - Adding an element increases length by one
    - If we assume bag semantics
  - Deleting an element that doesn't exist creates an error
  - If a list is empty, its length is 0
  - …

```
type list( T)
import
  integer, bool;
operators
  isEmpty:  list → bool;
  add:      list x T → list;
  contains: list x T → bool;
  delete:   list x T → list;
  length:   list → integer;
axioms: ∀ l: list, ∀ t: T
  length( add(l, t)) = length( l) + 1;
  length( l)=0 ⇔ isEmpty(l);
  …
```

List versus Points

```
type points
import
  coordinate, bool, list( coordinate);
Operators
  contains: points x coordinate → bool;
     # Implement as list.contains
  add:      points x coordinate → points;
     # Implement as list.add
  neighbor: points x coordinate → coordinate;
     # Not implemented in list!
axioms
  neighbor(p,c) = {x| contains(p,x)∧ ∀x':contains(p, x')=>
                                    distance(x,c) ≤ distance(x',c)};
```

- **`points`** uses a list and adds further functionality
- What‘s wrong?
  - What happens if multiple x have the same distance to c?

List versus Points

```
type points
import
  coordinate, bool, 2Dspace;
Operators
  contains: points x coordinate → bool;
  add:      points x coordinate → points;
  neighbor: points x coordinate → points;
axioms
  neighbor(p,c) = {x| contains(p,x) ∧ ∀x': contains(p,x'):
                                       distance(x,c) ≤ distance(x',c)};
```

Content of this Lecture

- Data Structures Again
- Abstract Data Types
- Example: Lists, Stacks, and Queues
  - Parameterized ADTs
- Realization in Java

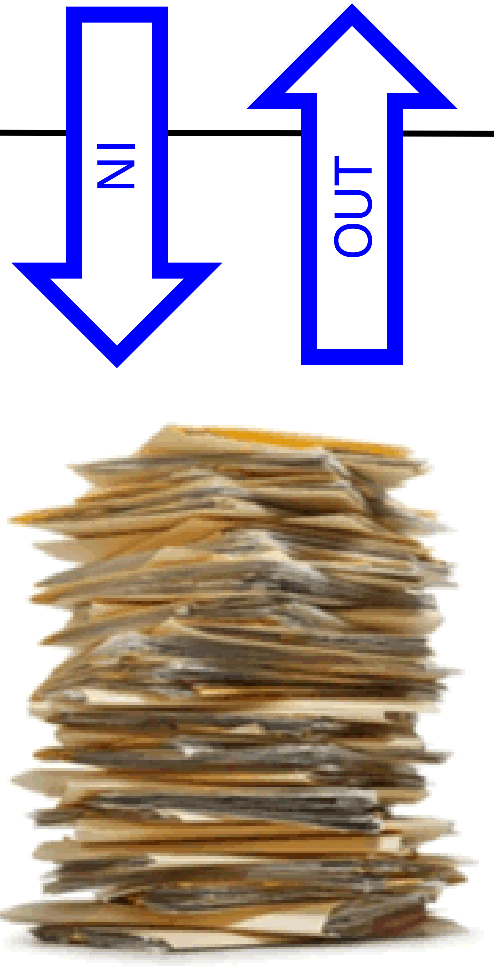
Lists, Stacks, Queues

- We looked at a data type (points, shops) which essentially is a list with one special operation: nearestNeighboor
  - Canonical list operations: insert, search, delete, update, length
- There are many ways to implement the general ADT **`list`**
  - Array, linked lists, double-linked lists, trees, ...
- Two types of lists are of exceptional importance in computer science: Stacks and Queues
  - Both support mostly two operations
  - These suffice for surprisingly many problems and applications
  - Can be implemented very efficiently

Queues

- Operations: enqueue, dequeue
- Special semantic: First in, first out (FIFO)
- Breadth-first traversal, shortest paths, BucketSort, ...

Stacks



- Operations: push, pop
- Special semantic: Last in, first out (LIFO)
- Call stacks, backtracking, “Kellerautomaten”, ...

As Abstract Data Types

```
type stack( T)
import
  bool;
operators
  isEmpty: stack → bool;
  push:    stack x T → stack;
  pop:     stack → stack;
  top:     stack → T;
```

```
type queue( T)
import
  bool;
operators
  isEmpty: queue → bool;
  enqueue: queue x T → queue;
  dequeue: queue → queue;
  head:    queue → T;
```

- Where‘s the difference?

Signature does not Suffice

```
type a( T)
import
  bool;
operators
  isEmpty: a → bool;
  add:     a x T → a;
  remove:  a → a;
  give:    a → T;
```

```
type a( T)
import
  bool;
operators
  isEmpty: a → bool;
  add:     a x T → a;
  remove:  a → a;
  give:    a → T;
```

- Where‘s the difference?
- From the signature alone, there is no difference
- Yet – we expect a different behavior

Defining the Difference

```
type stack( T)
import
  bool;
operators
  isEmpty: stack → bool;
  push:    stack x T → stack;
  pop:     stack → stack;
  top:     stack → T;
axioms ∀ q:stack, ∀ t:T
  top( push( s, t)) = t;
  pop( push( s, t)) = s;
```

```
type queue( T)
import
  bool;
operators
  isEmpty: queue → bool;
  enqueue: queue x T → queue;
  dequeue: queue → queue;
  head:    queue → T;
axioms ∀ q:queue, ∀ t:T
  head( enqueue(q, t)) =
    if isEmpty( q): t
    else head( q);
  dequeue( enqueue( q, t)) =
    if isEmpty( q): q
    else enqueue( dequeue(q), t);
```

Long version:

```
push(s,t) o top(s)=t' => t=t'
push(s,t) o pop(s)=s' => s=s'
```

Example

```
type queue( T)
...
  dequeue( enqueue( q, t)) =
    if isEmpty( q): q
    else enqueue( dequeue(q), t);
```

```
d( e( <3,2>, 5)) = e( d( <3,2>), 5) =
                   e( d( e( <3>, 2)), 5) =
                   e( e( d( <3>), 2), 5) =
                   e( e( d( e(<>), 3), 2), 5) =
                   e( e( <>, 2), 5) =
                   <2,5>
```

We Stop Here

- There are various ways to formally specify the behavior of operations of an ADT
- For instance: Algebraic specifications
  - Define an algebra over the object sets of the ADT
  - Includes axioms defining the semantics of operations
  - Axioms are essential to prove aspects of a system's behavior
    - Ideally, one only specifies and never programs
  - See lecture on "Modellierung und Spezifikation"
- In this lecture, we only look at signatures
  - No axioms
  - Supported by most programming languages (e.g. Java interfaces)

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ADTs in Java

- Recall
  - An ADT summarizes the essential operations on a set of objects
  - An ADT is independent of a realization/implementation
  - Any implementation of a ADT is called a concrete data type
- Realization in Java?
- Interfaces
  - Only exhibit the essential operations on a class of objects
  - Can have different implementations
  - Can be implemented by a concrete class

Remarks

- Java does not support axioms on interfaces
  - Some other languages do, e.g. contracts in Eiffel
- Java adds important functionality for practical work which we ignore in this lecture
  - Inheritance, visibility (public, protected, ...), overloading, ...
  - Critical: Encapsulation – you must not see anything of an object / do anything with an object that is not represented in its interface
  - See lectures on Software Engineering
- Historically, ADTs are a predecessor of classes in programming languages

Summary

- ADT's specify the possible operations on a data structure
- ADT's are free of implementation details
- We often discuss pros/contras of different ways to implement a given ADT
- We will often assume certain data types to be given
  - Always: Strings, integers, reals, ...
  - We make implicit assumptions on cost of operations: UCM
- (Formal) ADTs can be used for much more
  - Proving properties of a data type
  - Proving that a concrete data type implements a ADT
  - Proving that an implementation does not hurt axioms
  - Program verification

Exemplary Questions

- What is an abstract data type, what is a physical data type?
- What are typical operations of a list? Of a stack?
- Imagine a class storing rectangles in a plane. We want to add and remove rectangles, test if there are any rectangles, and find all rectangles intersection of given one. Define the ADT. What could be possible axioms?