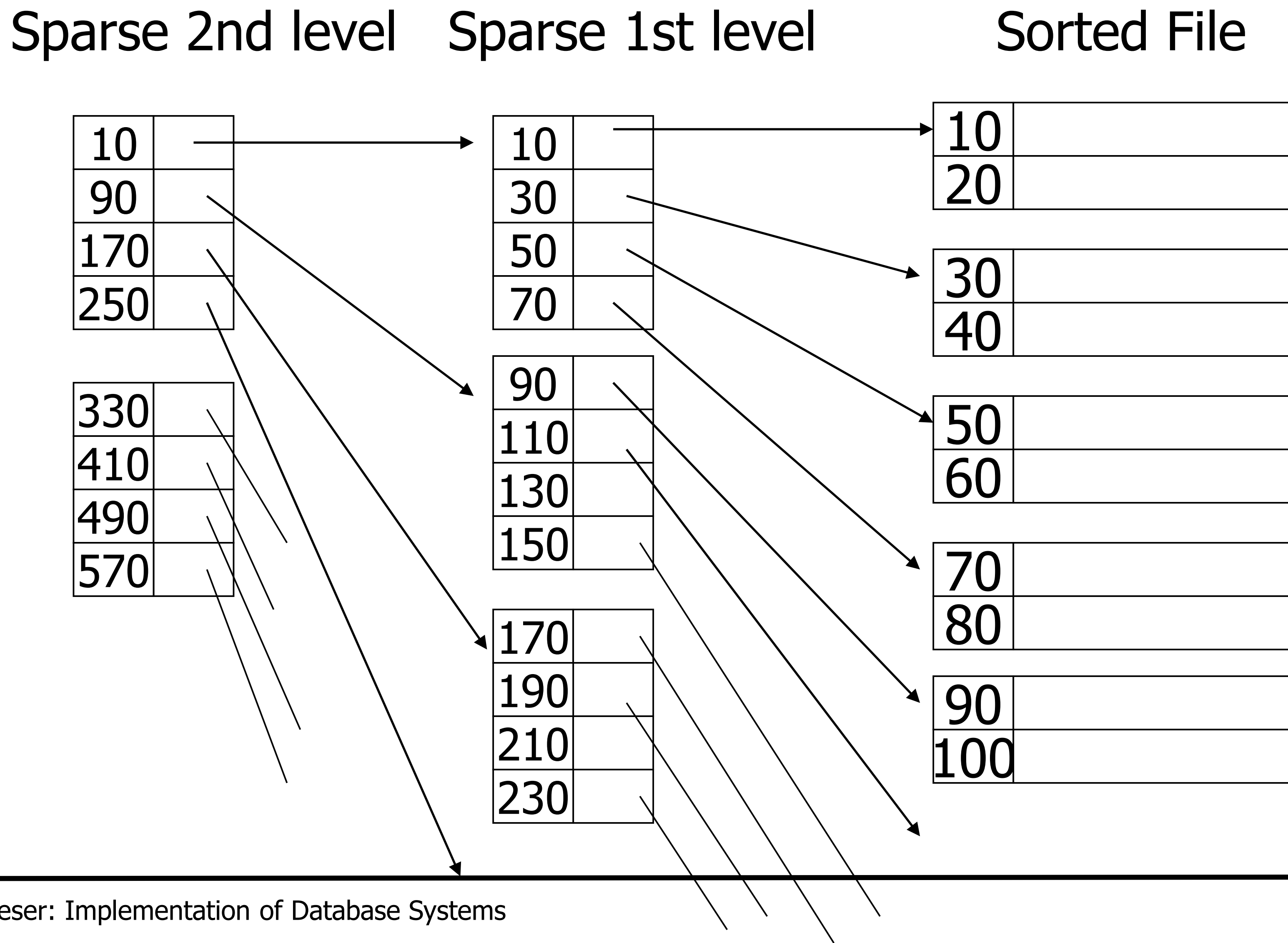
Datenbanksysteme II:
B / B+ / Prefix Trees

Ulf Leser

Content of this Lecture

- B Trees
- B+ Trees
- Index Structures for Strings

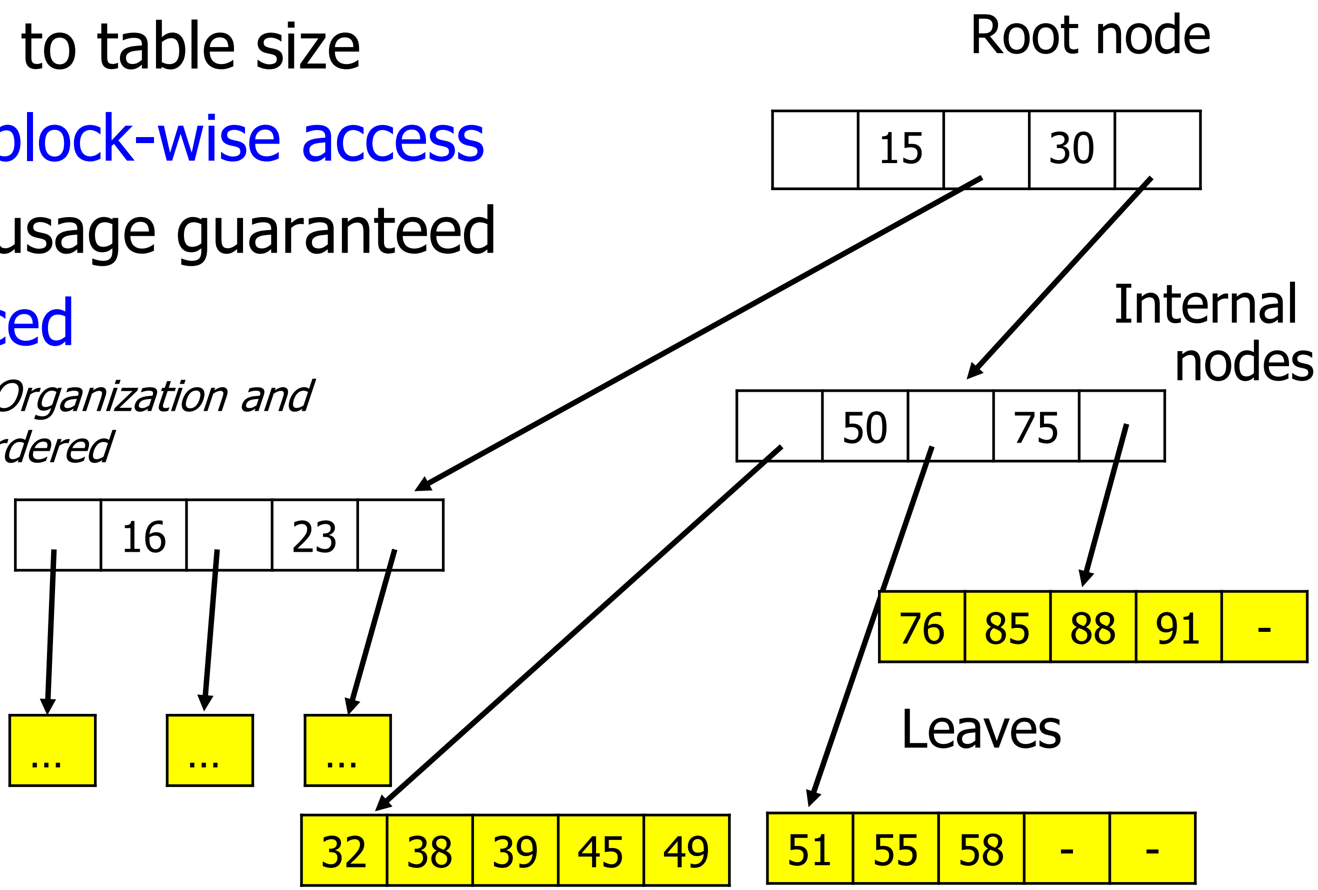
Recall: Multi-Level Index Files



B-Trees (≠ binary tree)

- B-Tree is a multi-level index with variable number of levels
  - Many variations: B/B+/B*/B++/BB…
- Height adapts to table size
- Designed for block-wise access
- >50% space usage guaranteed
- Always balanced

R. Bayer, E. McCreight: *Organization and Maintenance of Large Ordered Indexes. Acta Informatica.* 1972

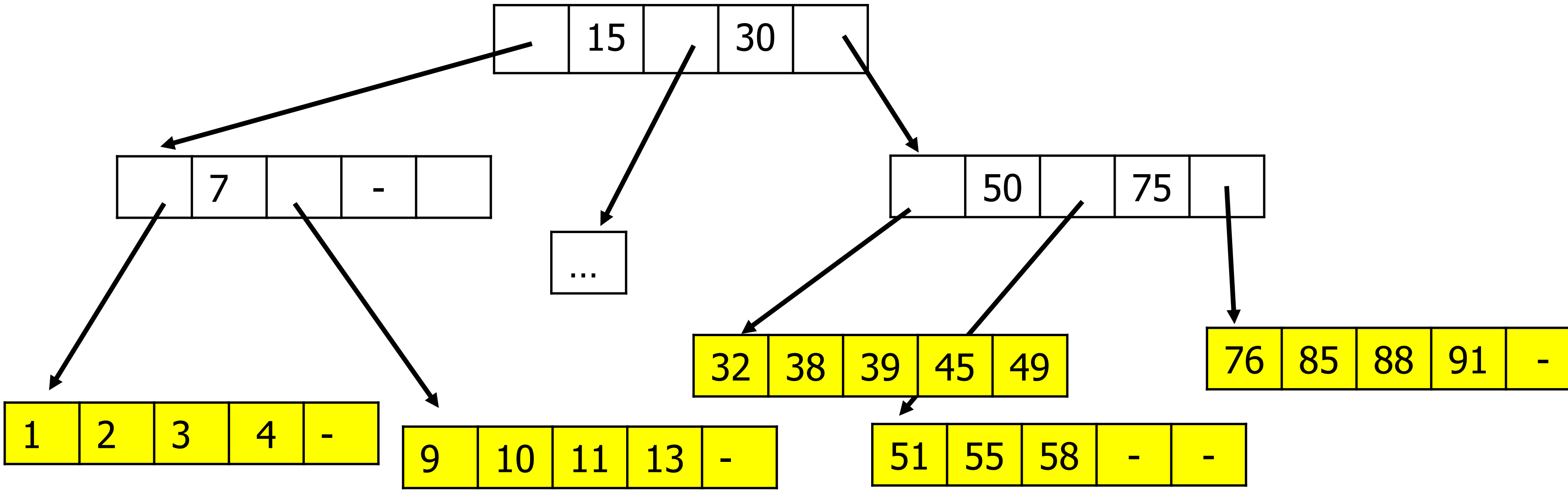


Formally

- Assume index on primary key (no duplicates)
- Internal nodes contain pairs (key, TID) and pointers
- Leaf nodes only contain (key, TID)
- Block can hold 2k triples (pointer, key, TID) plus 1 ptr
- Each internal node contains between k and 2k (key, TID)
  - And between k+1 and 2k+1 pointers to subtrees
    - Subtree left of pair (v,TID) contains only and all keys y<v
    - Subtree right of pair (v,TID) contains only and all keys y>v
    - Pairs are sorted: $v_i < v_{i+1}$
  - Exception: Root node
- Thus, B-trees use always at least 50% of allocated space

p_0	(v_0,t_0)	p_1	(v_1,t_1)	p_2	(v_2,t_2)	p_3	…	(v_{2k-1},t_{2k-1})	p_{2k}

Searching B-Trees



Find 9

1. Start with root node
2. Follow p_0
3. Follow p_1
4. Scan block - found

Find 60

1. Start with root node
2. Follow p_2
3. Follow p_1
4. Scan block - not found

Complexity

- B-trees are always balanced (how? Wait)
  - All paths from root to a leaves are of equal length
- Assume n keys; let r=|key|+|TID|+|pointer|
- Best case: All nodes are full (2k keys)
  - We have b~n/2k blocks
    - Actually a little less, since leaves contain no pointers
  - Height of the tree $h \sim \log_{2k}(b)$
  - Search requires between 1 and $\log_{2k}(b)$ IO
- Worst case: All nodes contain only k keys
  - We need b~n/k blocks
  - Height of the tree $h \sim \log_k(b)$
  - Search requires between 1 and $\log_k(b)$ IO

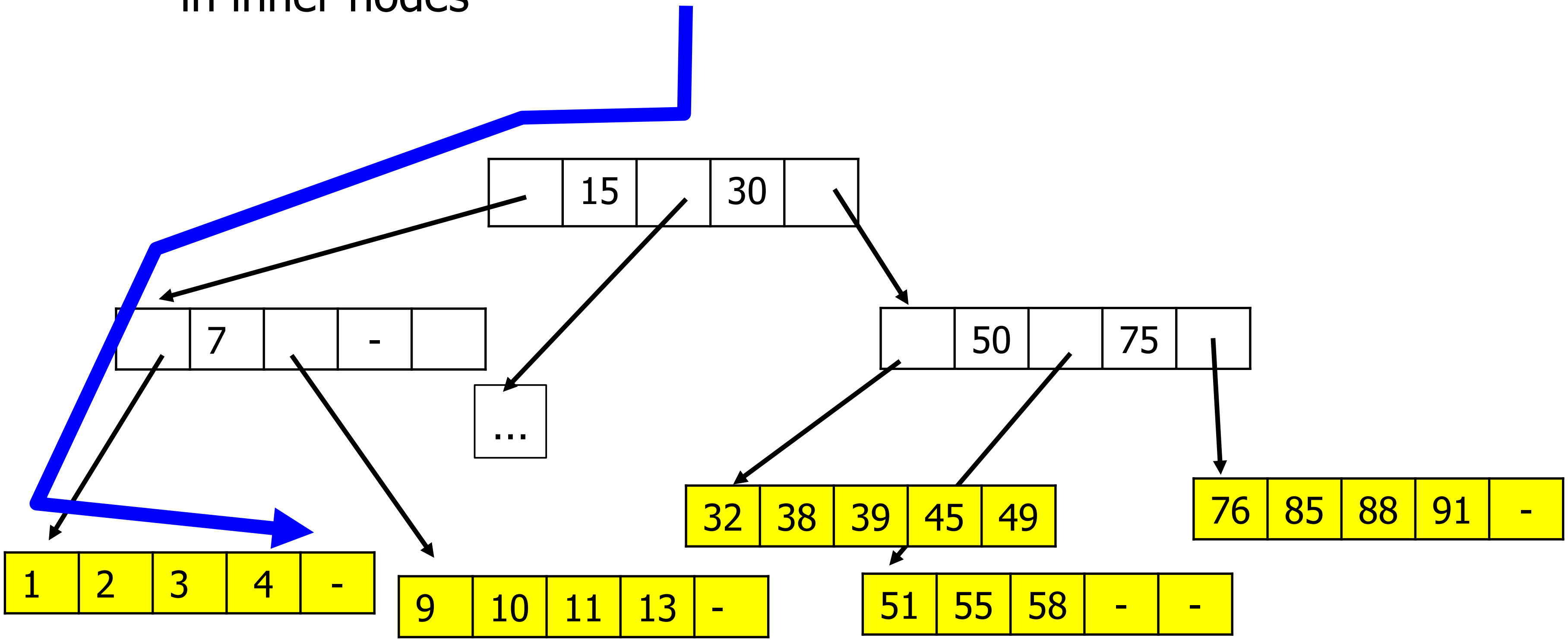
Example

- Assume |key|=20, |TID|=16, |pointer|=8, block size=4096 => r=44
- Assume n=1.000.000.000 (1E9) records

- Gives between 46 and 92 index records per block
- Hence, we need 5 or 6 IO
  - Essentially all data is in the leaves
  - Very small changes to find key earlier
- Caching the first two levels (between 1+46 and 1+92 blocks), this reduces to 3 or 4 IO

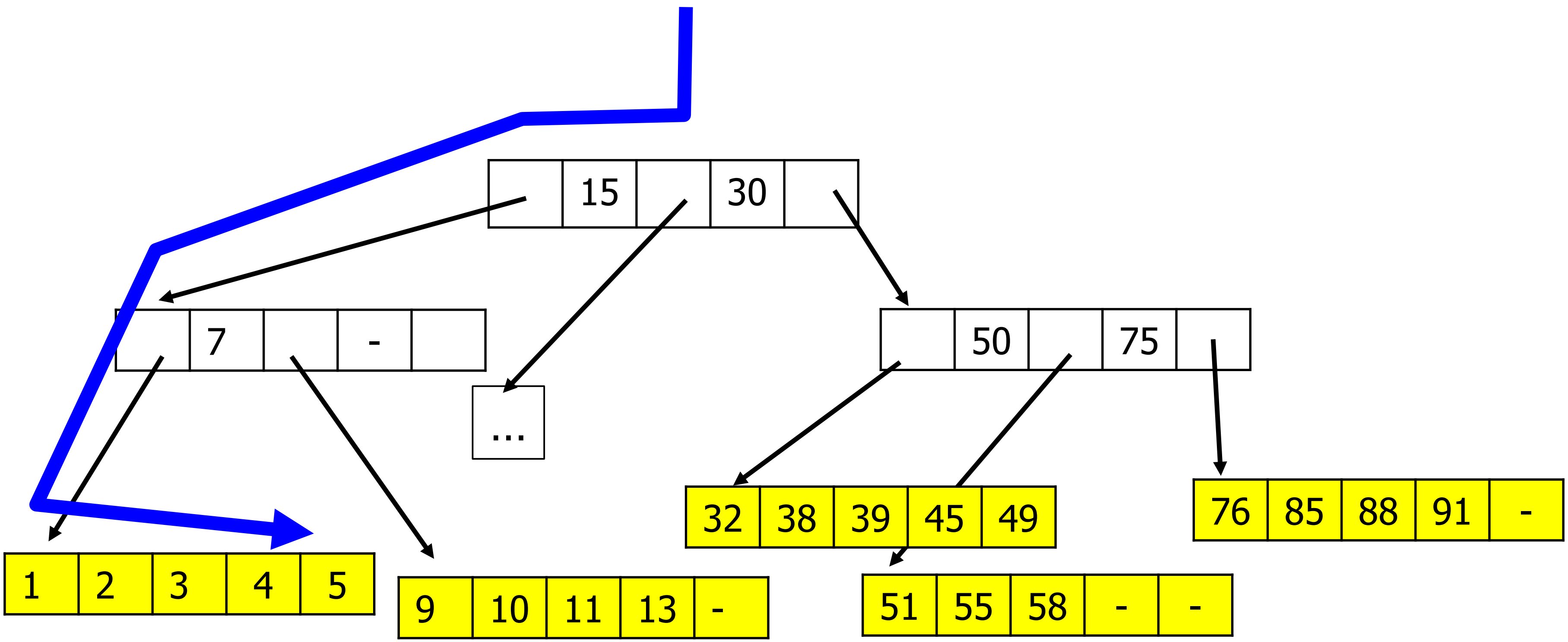
Inserting into B-Trees

- In B-Trees, we always insert into a leaf
- We insert 5 (assume: 2*k=2)
  - For ease of exposition, we assume 2-5 keys in leaves and 1-2 keys in inner nodes



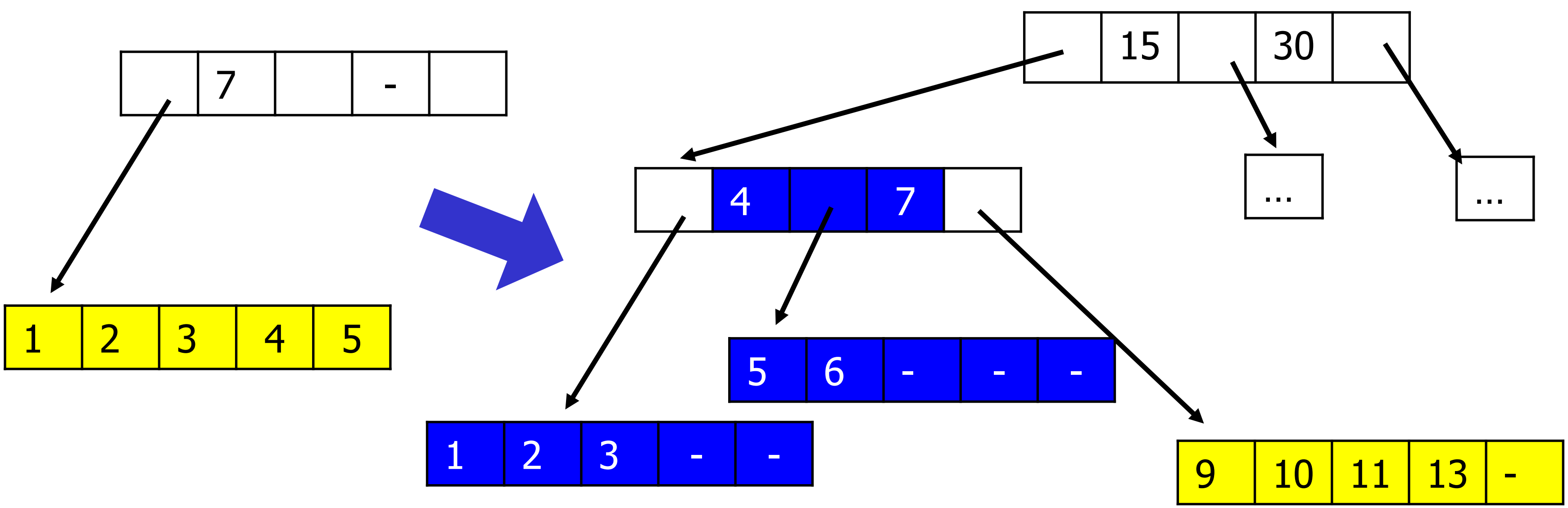
Inserting into B-Trees

- We insert 6
- Block is full – we need to split



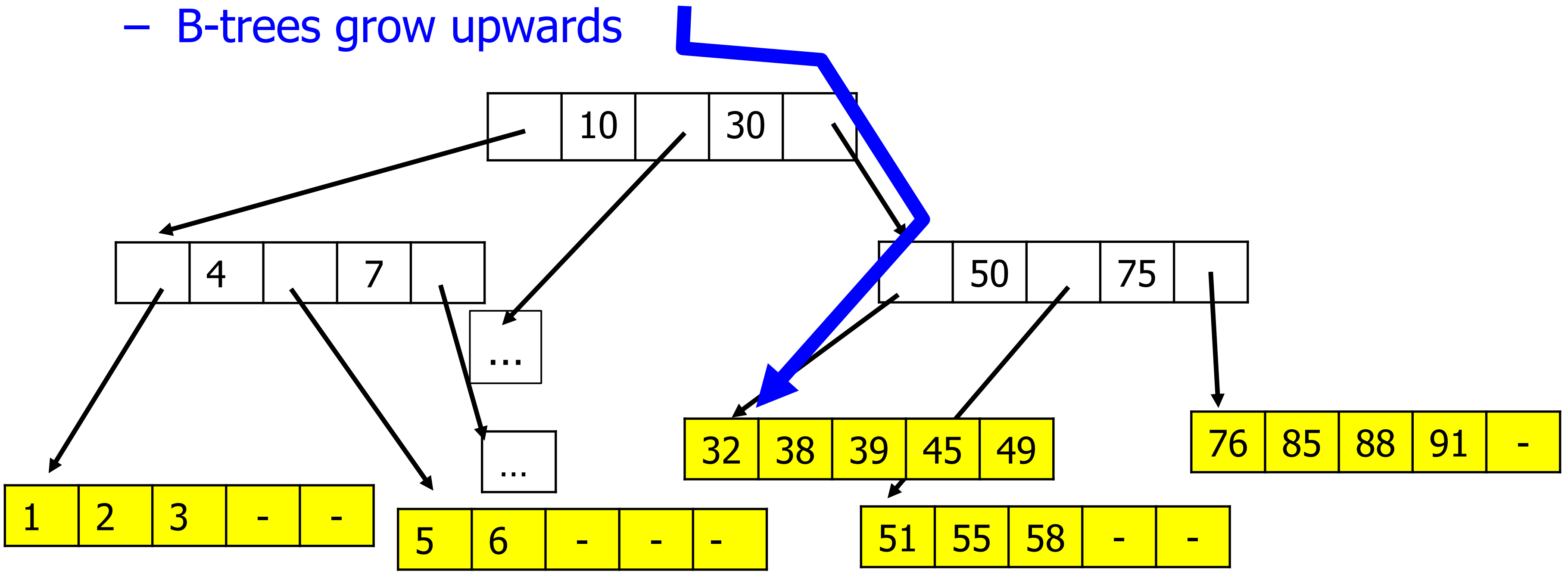
Inserting into B-Trees

- Split overflow block and propagate median upwards
  - All values from old node plus new value minus median are evenly split between two new nodes
  - Thus, each has ~k keys
  - Median is pushed up to parent node and inserted there

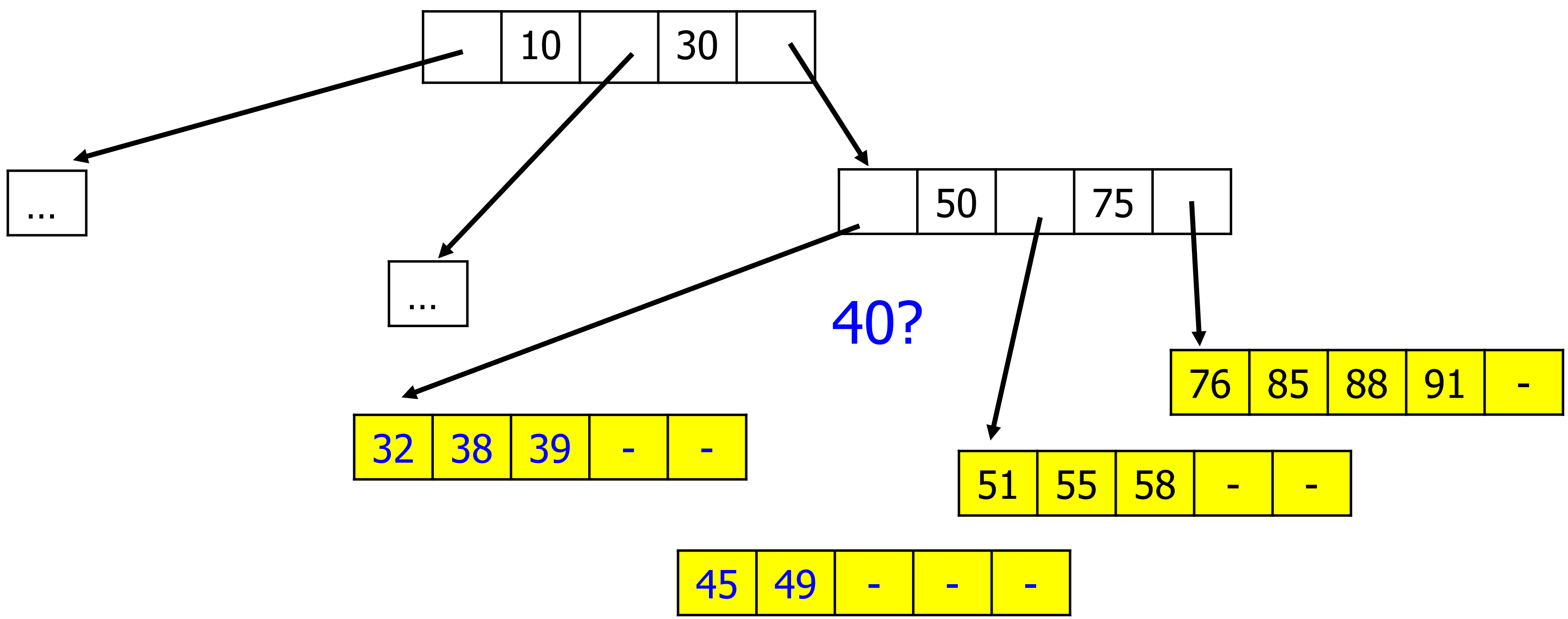


Inserting into B-Trees

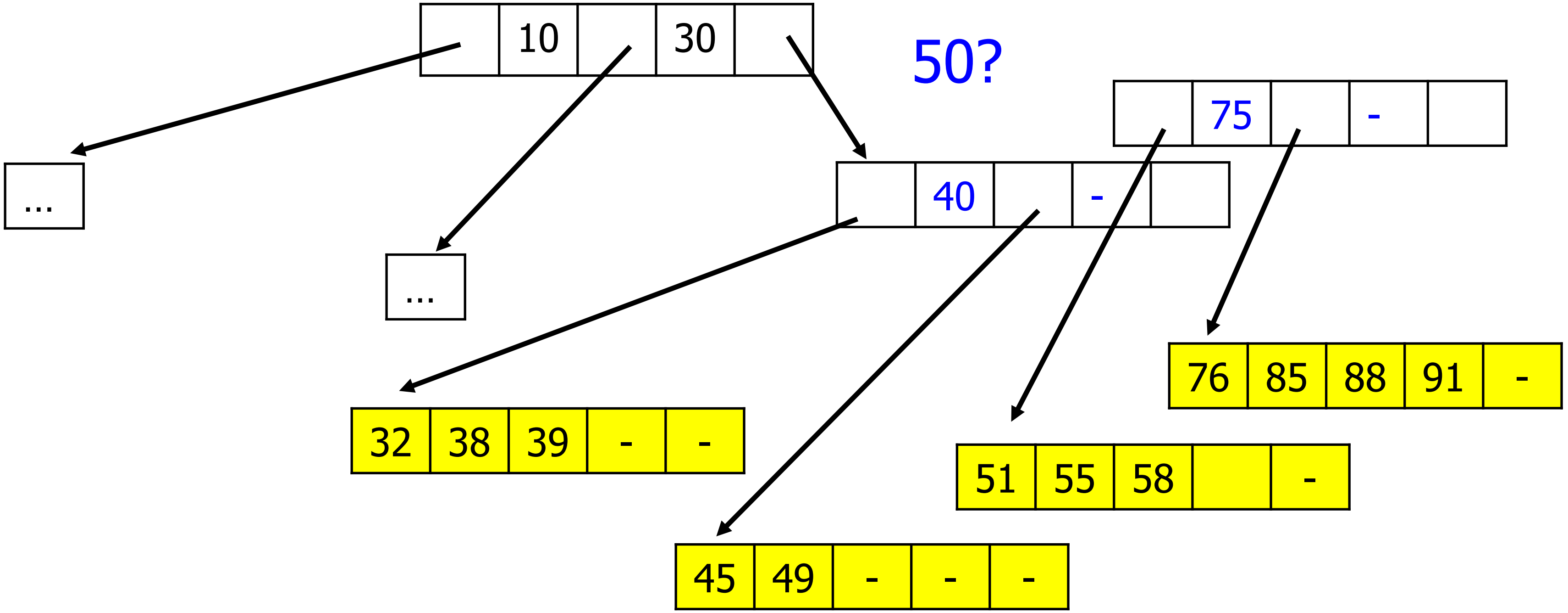
- We insert 40
- Block is full – split and propagate 40, the median
- Propagating upwards leads to overflow in parent(s)
- Finally, the root note overflows
  - B-trees grow upwards



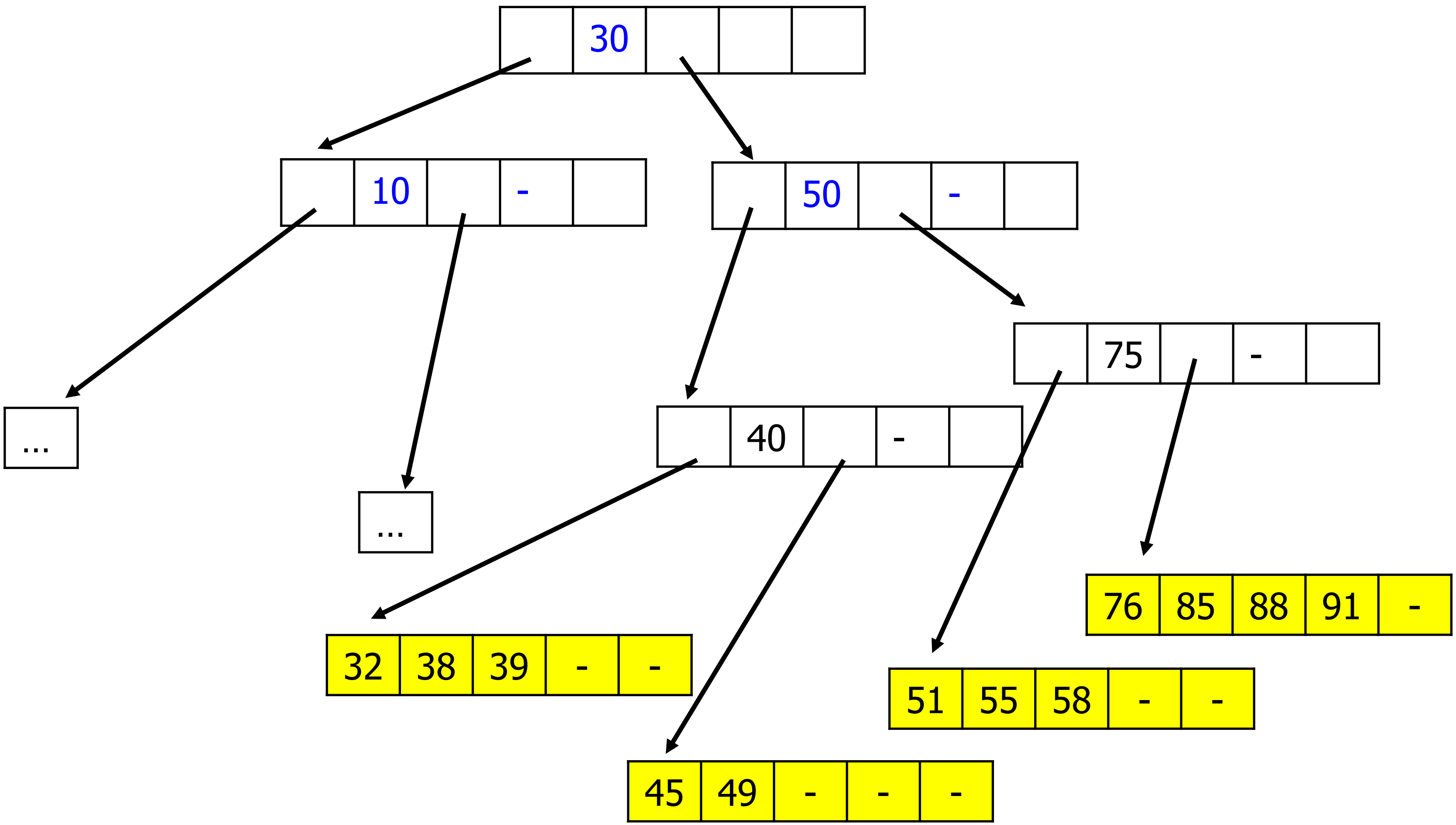
Intermediate 1



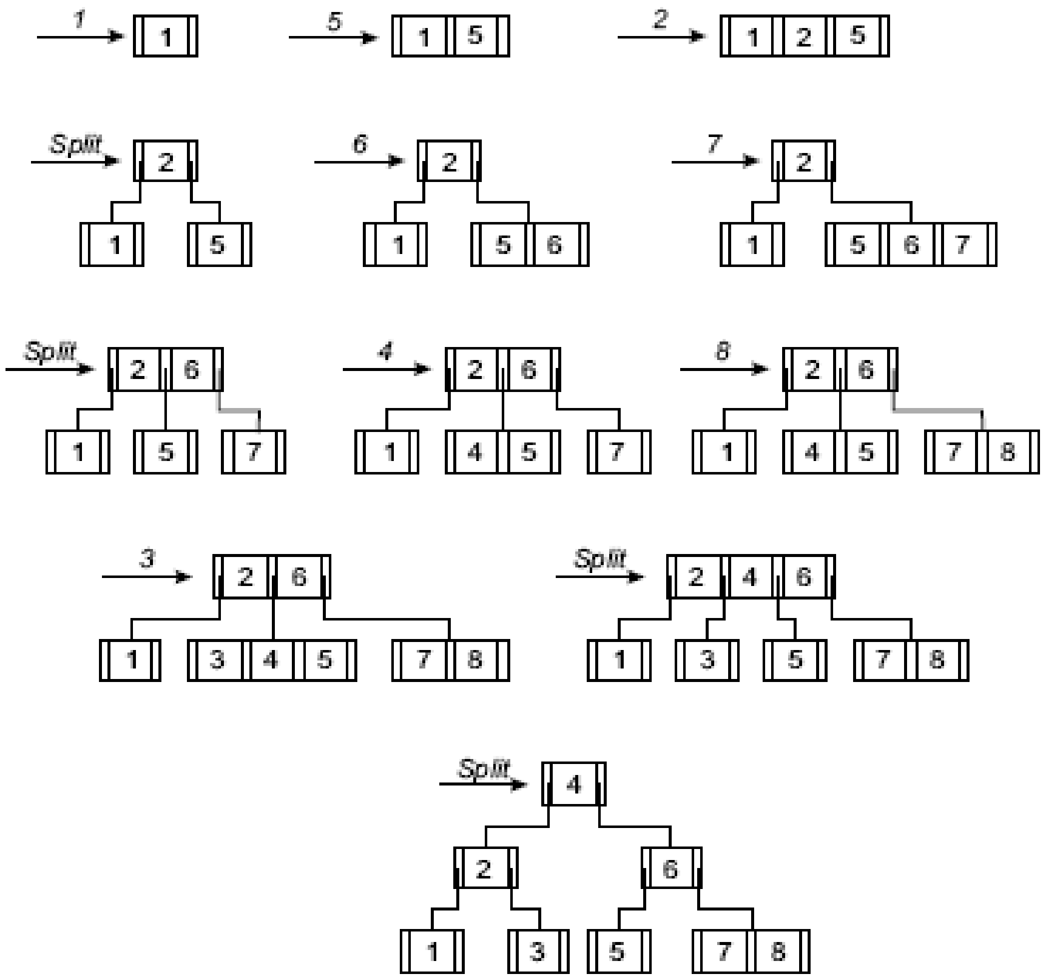
Intermediate 2



Final Tree



Longer Sequence of Insertions



Complexity Insertion

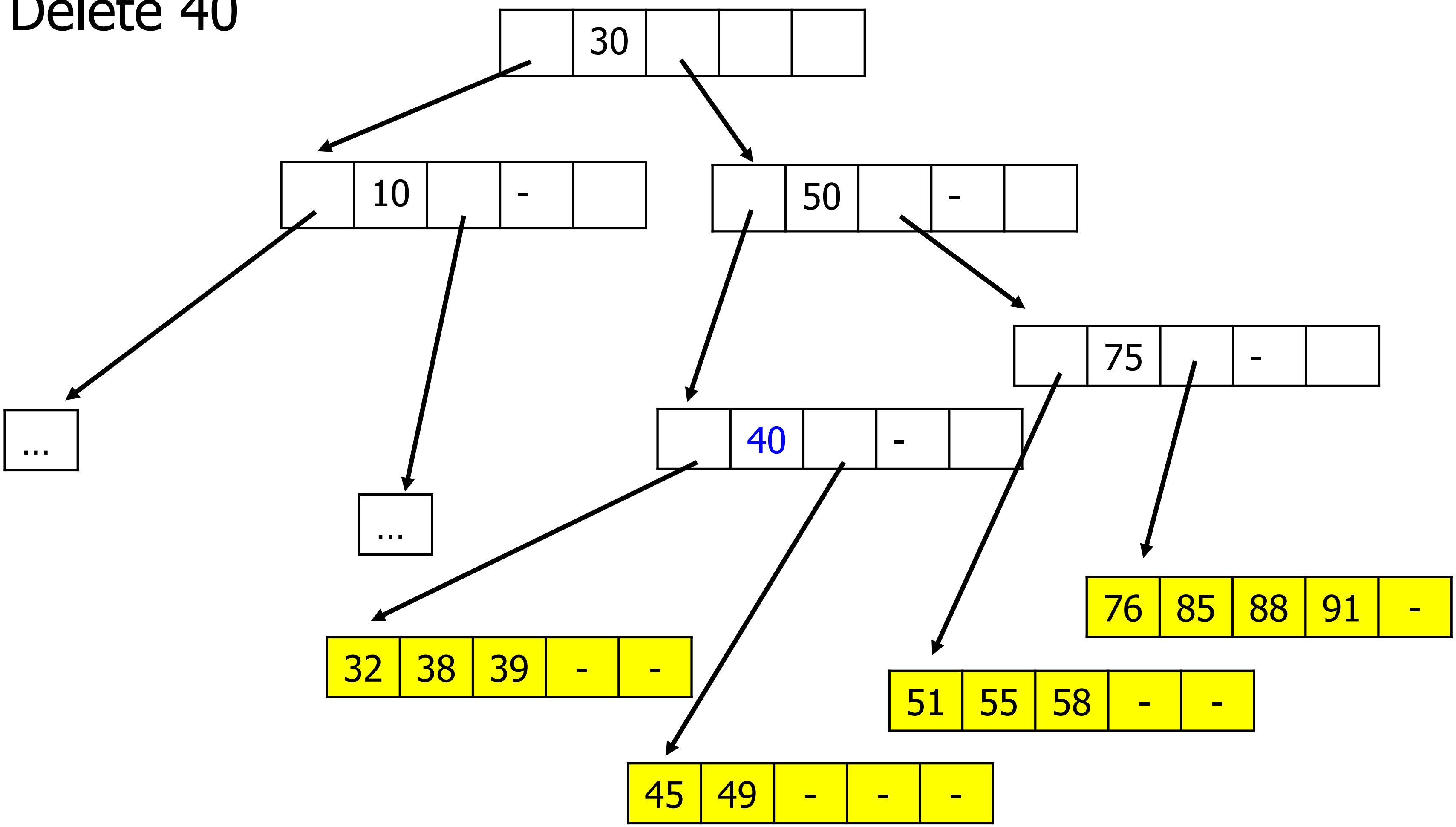
- Let h be height of B-tree
- Cost for searching leaf node: h IO
- If no split necessary: Total IO cost = h+1 (writing)
- If split is necessary
  - Worst case – up to the root
  - We assume we cached ancestor blocks during traversal
  - We thus need to read them once and write them once
  - Total cost: (h+2)+2(h-1)+1 = 3h+1
    - Split on all levels and create new root node

Deleting Keys

- If found in internal node
  - Choose smallest value from right subtree and replace deleted value
    - This value must be in a leaf
    - Recall search trees: symmetric predecessor (or successor)
  - Delete value in leaf and progress
- If found in leaf
  - Delete value
  - If blocks underflows (<k keys), choose one of neighboring blocks
    - Must have the same parent node
  - If both blocks together have more than 2k records: Distribute values evenly; adapt between-key in parent node
  - Otherwise – merge blocks
    - One block with all leaf-records plus the median in parent
    - Remove middle value in parent block – which now might underflow
  - Might work recursively up the tree

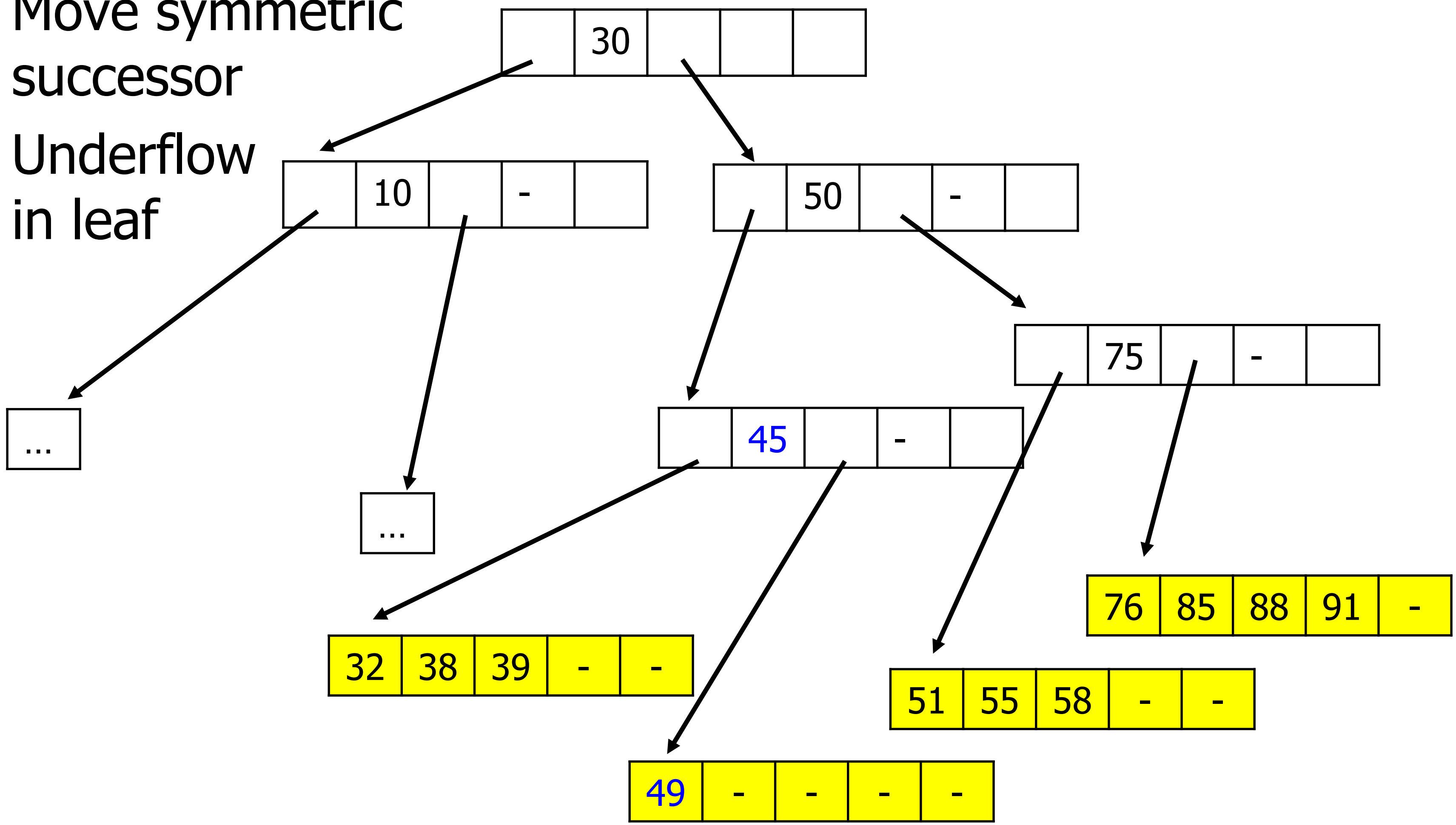
Delete with Underflow

- Delete 40



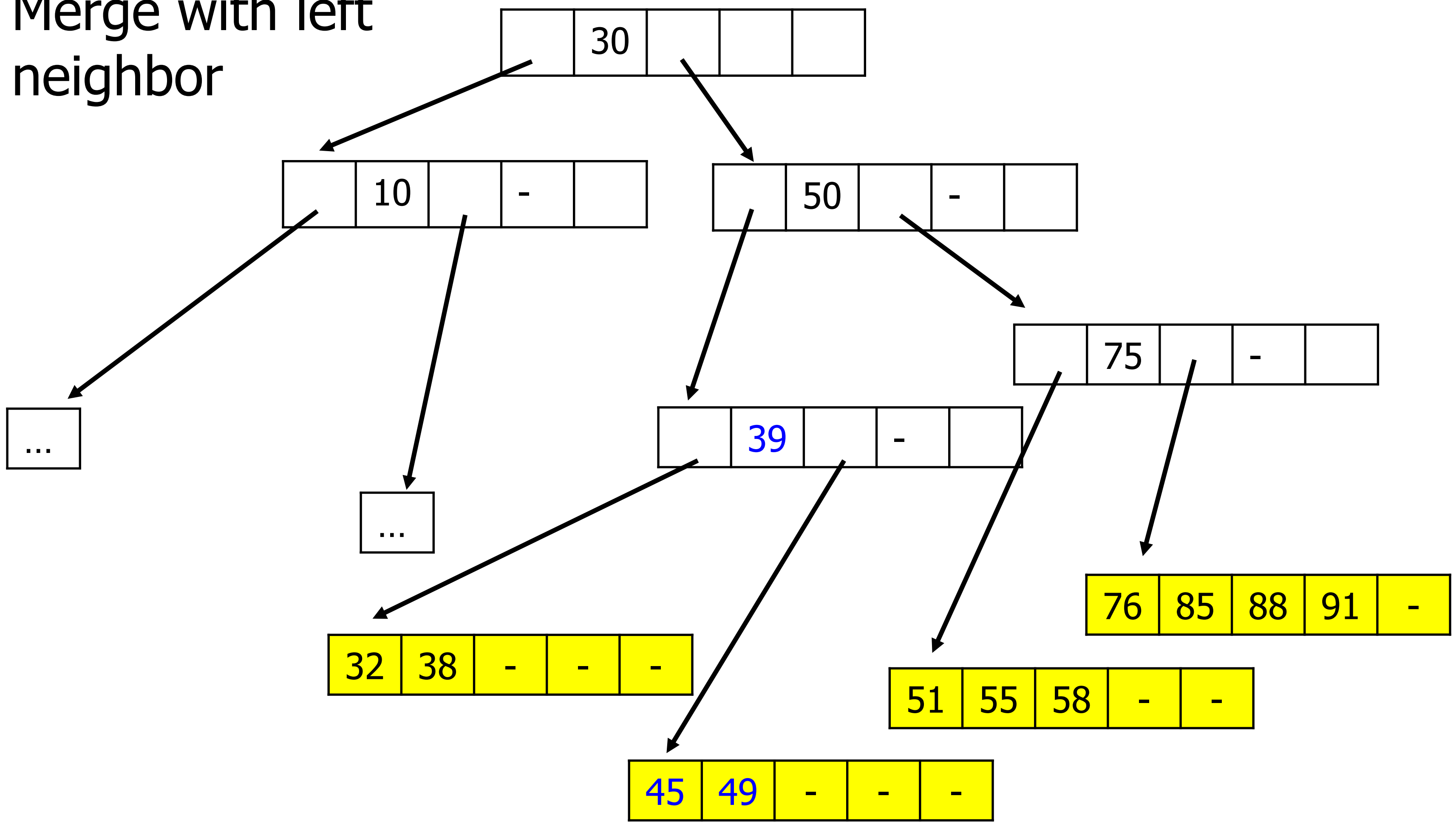
Delete with Underflow

- Move symmetric successor
- Underflow in leaf



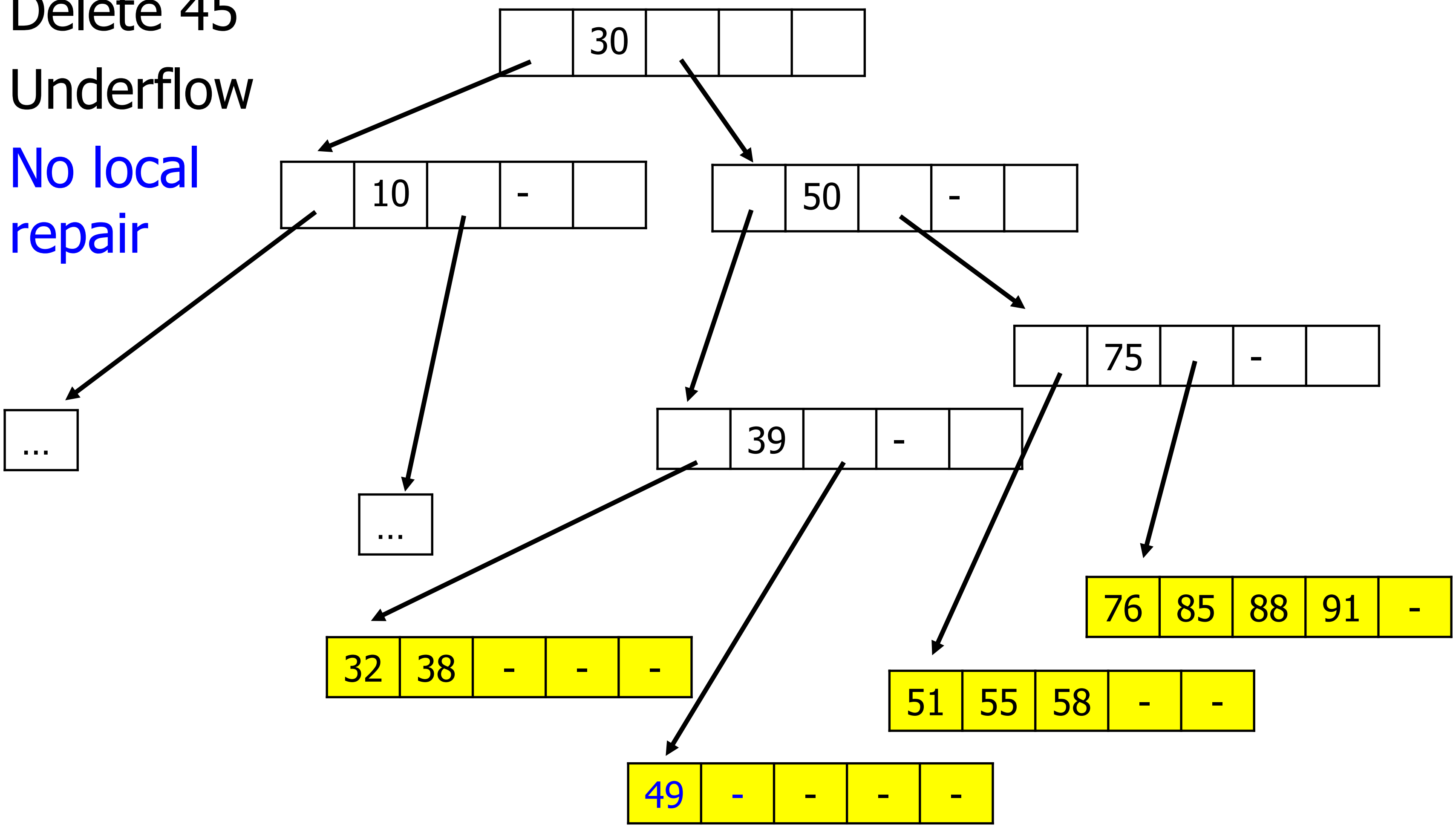
Delete with Underflow

- Merge with left neighbor



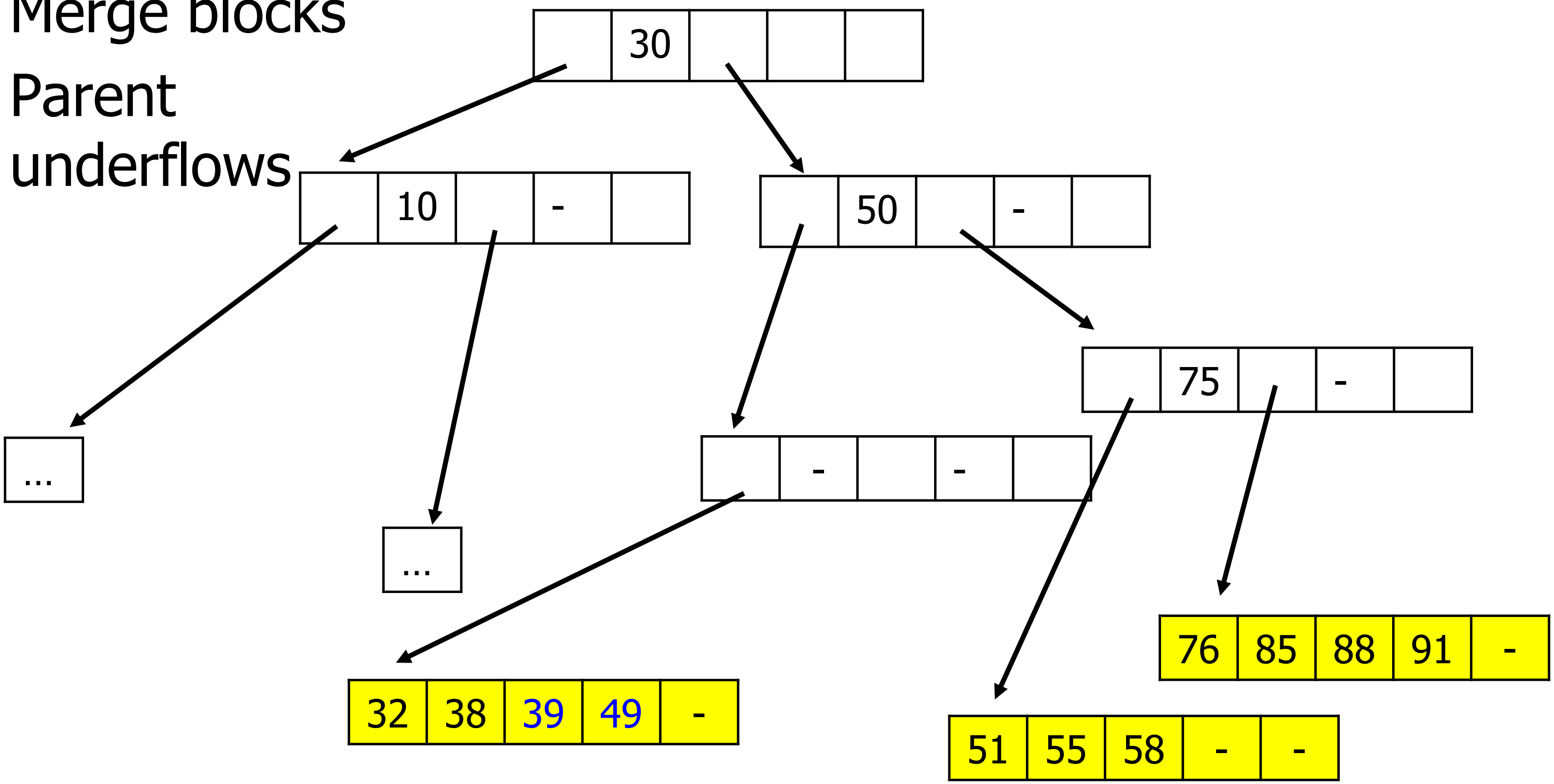
Delete with Underflow

- Delete 45
- Underflow
- No local repair



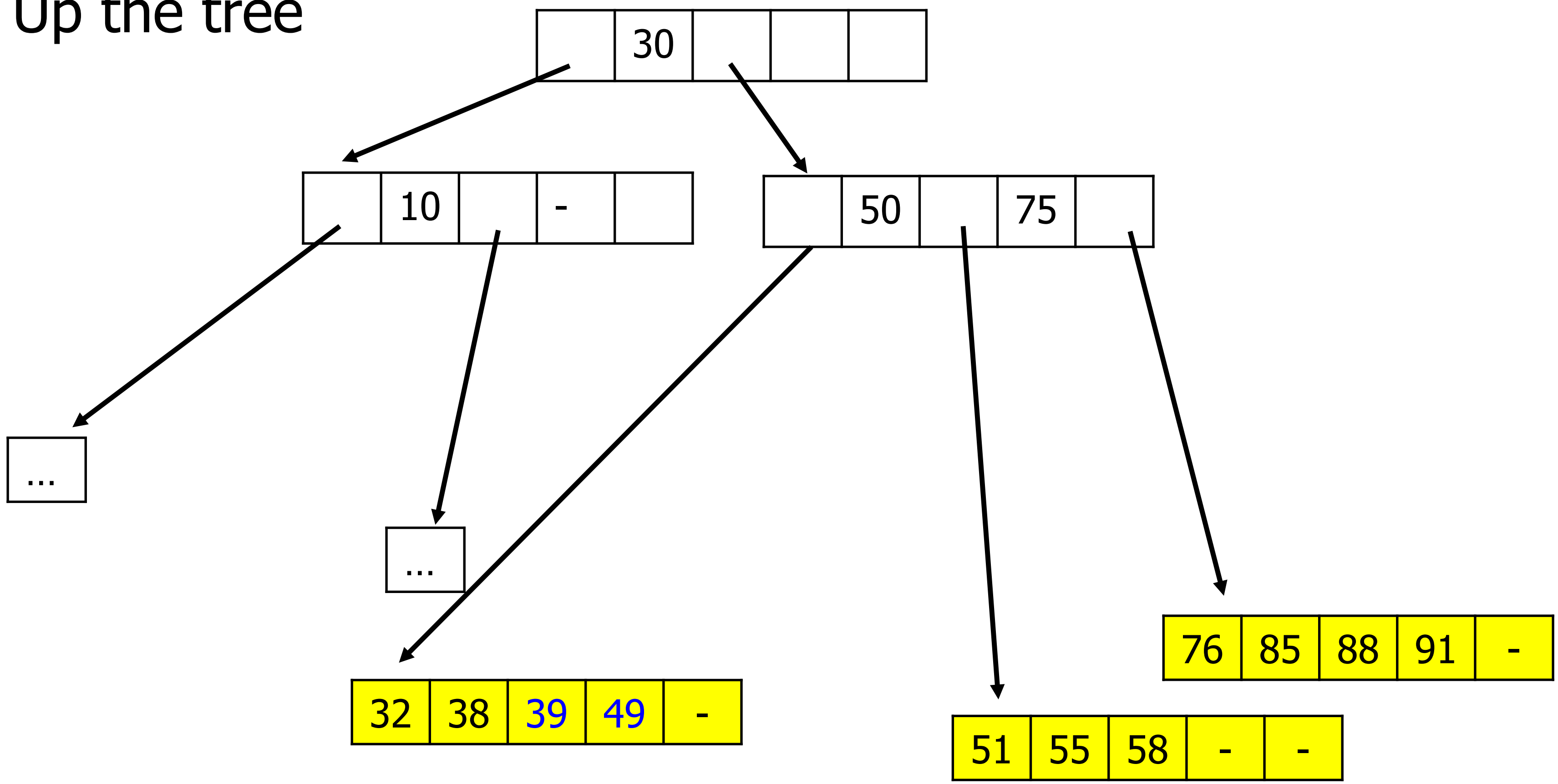
Delete with Underflow

- Merge blocks
- Parent underflows



Delete with Underflow

- Up the tree



Complexity of Deleting Keys

- Going down costs h IO
  - If key found in leaf, it costs h to read and 1 to write
  - If found in internal node, we still have to read h blocks to choose replacement value from leaf
- If no underflow, total cost is h+1
- If local underflow (with merge), total cost is ~h+4
  - Checking left and right neighbor, writing block and chosen neighbor, writing parent
- If blocks underflow bottom-up, total cost is at most 4h-2
  - If left and right neighbors have to be checked at each level

B-trees on Non-Unique Attributes

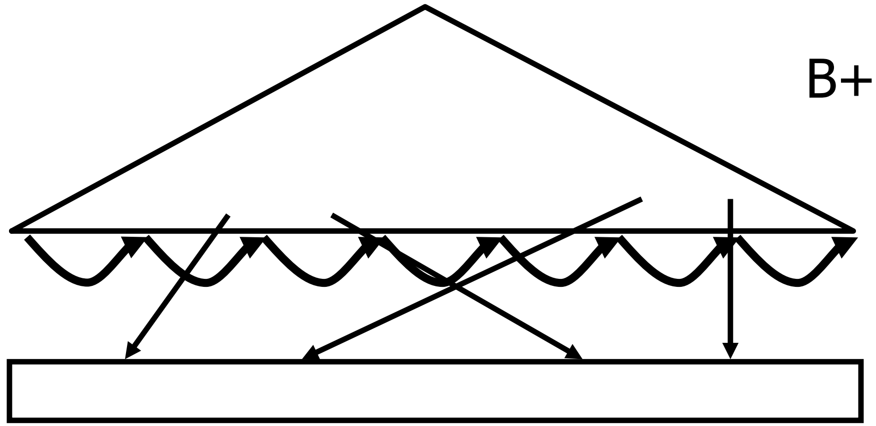
- If duplicates exist
- Option 1: Compact representation
  - Store (value, TID_1, TID_2, ... TID_n)
  - Difficult– internal nodes don't have fixed number of pairs any more
  - Requires internal overflow blocks
- Option 2: Verbose representation
  - Treat duplicates as different values
  - Constraints on keys change from "<" to "≤"
  - Extreme case: Generates a tree although a list would suffice
- Better: B+ trees

Content of this Lecture

- B Trees
- B+ Trees
- Index Structures for Strings

B+ Trees

- Dense index on heap-structured data file
- Internal nodes contain only values and pointers
  - Values demark borders between subtrees
  - Concrete values need not exist as keys - only signposts
- Leaves are chained for faster range queries



B+ Tree as dense index

Data file organized as heap file

Operations

- Searching
  - Essentially the same as for B trees
  - But will always go down to leaf – marginally worse IO complexity
- Insertion
  - Essentially the same as for B trees
  - When block is split, no value moves upwards
    - Parent block still changes – new signpost
    - Typical choice: avg($v_{median-1}$, $v_{median+1}$)
- Deletion
  - Deletion in internal node cannot occur
  - When blocks are merged, no values are moved up
    - But signposts in parent node are deleted as well

Advantages

- Simpler operations
- Higher fan-out, lower IO complexity
  - No TIDs in internal nodes - more pointers in internal nodes
  - Much reduced height (base of log() changes)
- Smoother balancing: Chose signposts carefully
  - Choose such that future inserts are evenly distributed
- Linked leaves
  - Faster range queries – traversal need not go up/down the tree
  - Optimally, leaves are in sequential order on disk

B* tree: Improving Space Usage

- Can we increase space usage guarantee beyond 50%?
- Don‘t split upon overflow: Move values to neighbor blocks as long as possible
  - More complex operations, need to look into neighbors
  - We only split when all neighbors and the current block is full
- When splitting, make three out of two
  - We only split when all neighbors are full – choose one
  - Generate three new blocks from the two full old ones
  - Each new block has 4/3k keys: Guaranteed 66% space usage

- Knuth, D. E.: *The Art of Computer Programming, Volume III: Sorting and Searching Addison-Wesley*, 1973

B+ Trees and Hashing

- Hashing faster for some applications
  - Can lead to O(1) IO
  - Assumes good hash function
  - Requires domain knowledge
- B+ trees
  - Very few IO if upper levels are cached
  - Adapts to skewed (non-uniformly distributed) data
  - Domain-independent
  - Also supports range queries

Loading a B+ Tree

- What happens in case of

```
create index myidx on LARGETABLE( id);
```

Loading a B+ Tree

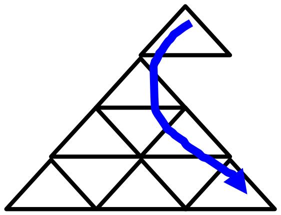
- What happens in case of

```
create index myidx on LARGETABLE( id);
```

- Naïve: Record-by-record insertion
  - Each insertion has 3h+1 = $O(\log_k(b))$ block IO
  - Altogether: $O(n*\log_k(b))$
- Blocks are read and written in arbitrary order
  - Very likely: bad cache-hit ratio
- Space usage will be anywhere between 50 and 100%
- Can't we do better?

Bulk-Loading a B+ Tree

- First sort records
  - $O(n*\log_m(n))$, where m is number of records fitting into memory
  - Clearly, m>>k
- Insert in sorted order using normal insertion
  - Tree builds from lower left to upper right
  - Caching will work very well
  - But space usage will be only around 50%



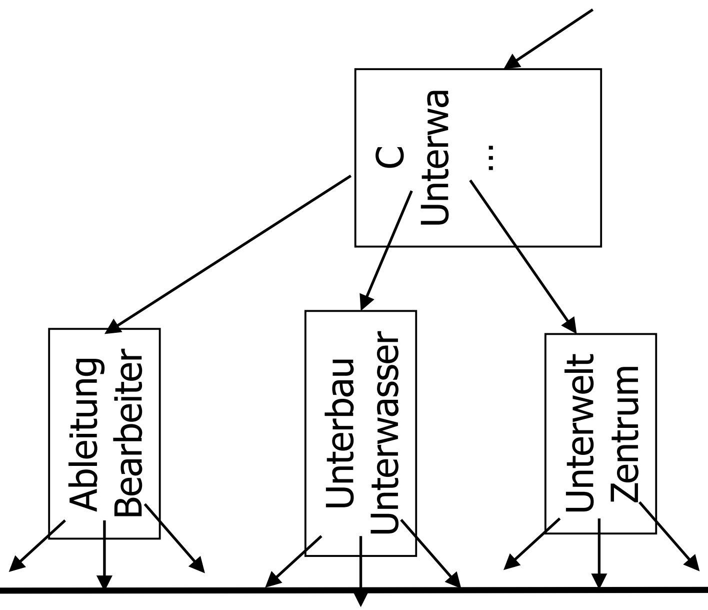
- Alternative
  - Compute structure in advance
    - Every 2k'th record we need a separating key
    - Every 2k'th separating key we need a next-level separating key
    - …
  - Can be generated and written in linear time

Content of this Lecture

- B Trees
- B+ Trees
- Index Structures for Strings
  - Prefix B+ Tree
  - Prefix Tree
  - PETER
  - PEARL

Prefix B+ Trees

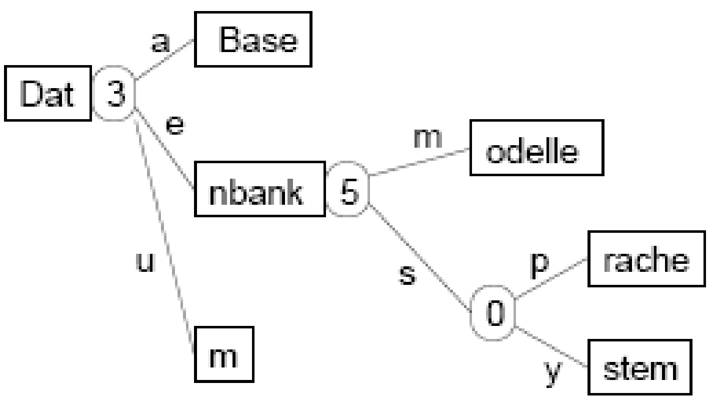
- Consider string values as keys
- Keys for int. nodes: Smallest key from right-hand subtree
  - Leads to internal signposts as large as keys
- Prefix B+ trees – Shortest string separating largest key in left-hand subtree from smallest key in right-hand subtree



Advantages:	Reduced space, higher fan-out
Disadvantages:	Overhead for computing signpost

Prefix Tree / Patricia tree / Trie

- If we index many strings with many common prefixes
  - … as in Information Retrieval …
  - Why store common prefixes multiple times?
- Prefix trees
  - Store common prefix / substring in internal nodes
  - Searching a key k requires at most |k| character comparisons



Indexing Strings

- Prefix trees traditionally are main memory structures
  - How to optimally layout internal nodes on blocks?
  - Not balanced – no guaranteed worst-case IO
- More index structures for strings
  - Keyword trees – searching for many patterns simultaneously
    - Necessary for joins on strings
    - Persistent keyword trees – challenge
  - Suffix trees – indexing all substrings of a string
    - Necessary e.g. to search genomic sequences
    - Persistent suffix trees – challenge in advancement

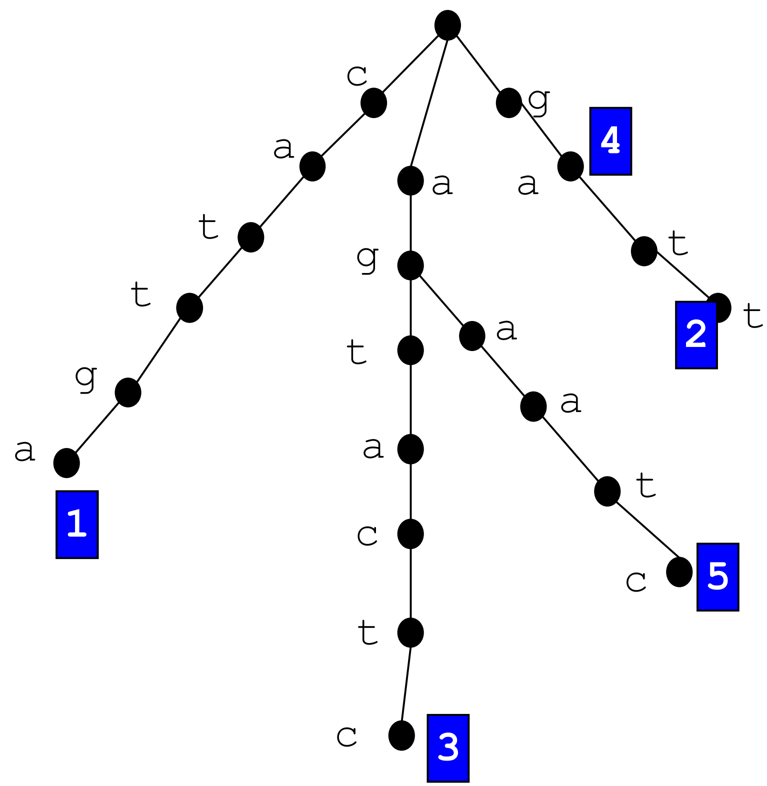
PETER

- Computes joins / search on large collections of long strings much faster than traditional DB technology
- Also handles similarity search / similarity joins
- Open source
  - Rheinländer, A., Knobloch, M., Hochmuth, N. and Leser, U. (2010). "Prefix Tree Indexing for Similarity Search and Similarity Join on Genomic Data". SSDBM 2010
- There are many similar index structures
  - PRETTY, PRETTY+, MASSJoin, …

Prefix-Trees

- Given a set S of strings
- Build a tree with
  - Labeled nodes
  - Outgoing edges have different label
  - Every s∈S is spelled on exactly one path from root
  - Mark all nodes where a string ends
- Common prefixes are represented only once

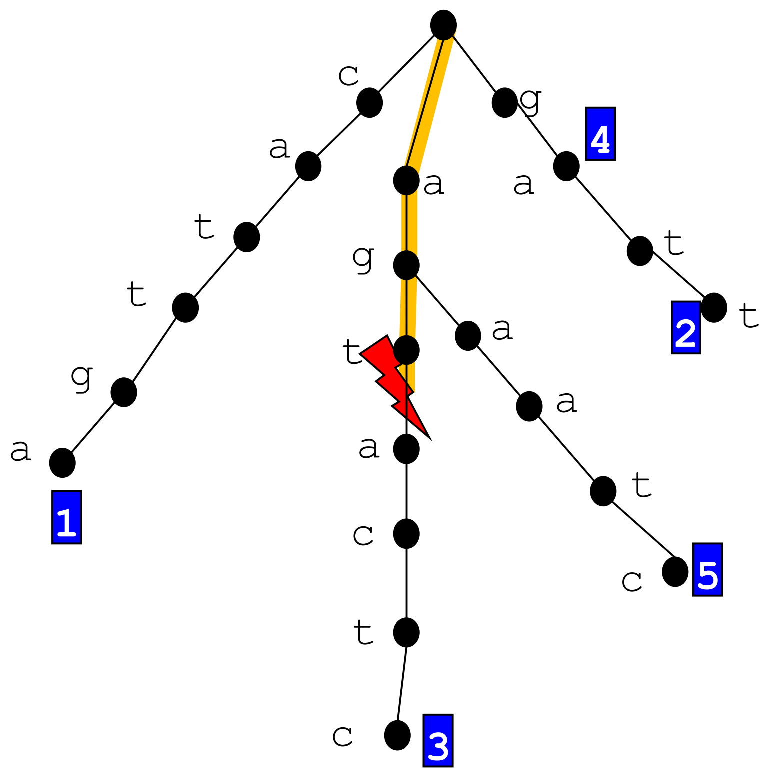
cattga, gatt, agtactc, ga, agaatc



Searching Prefix-Trees

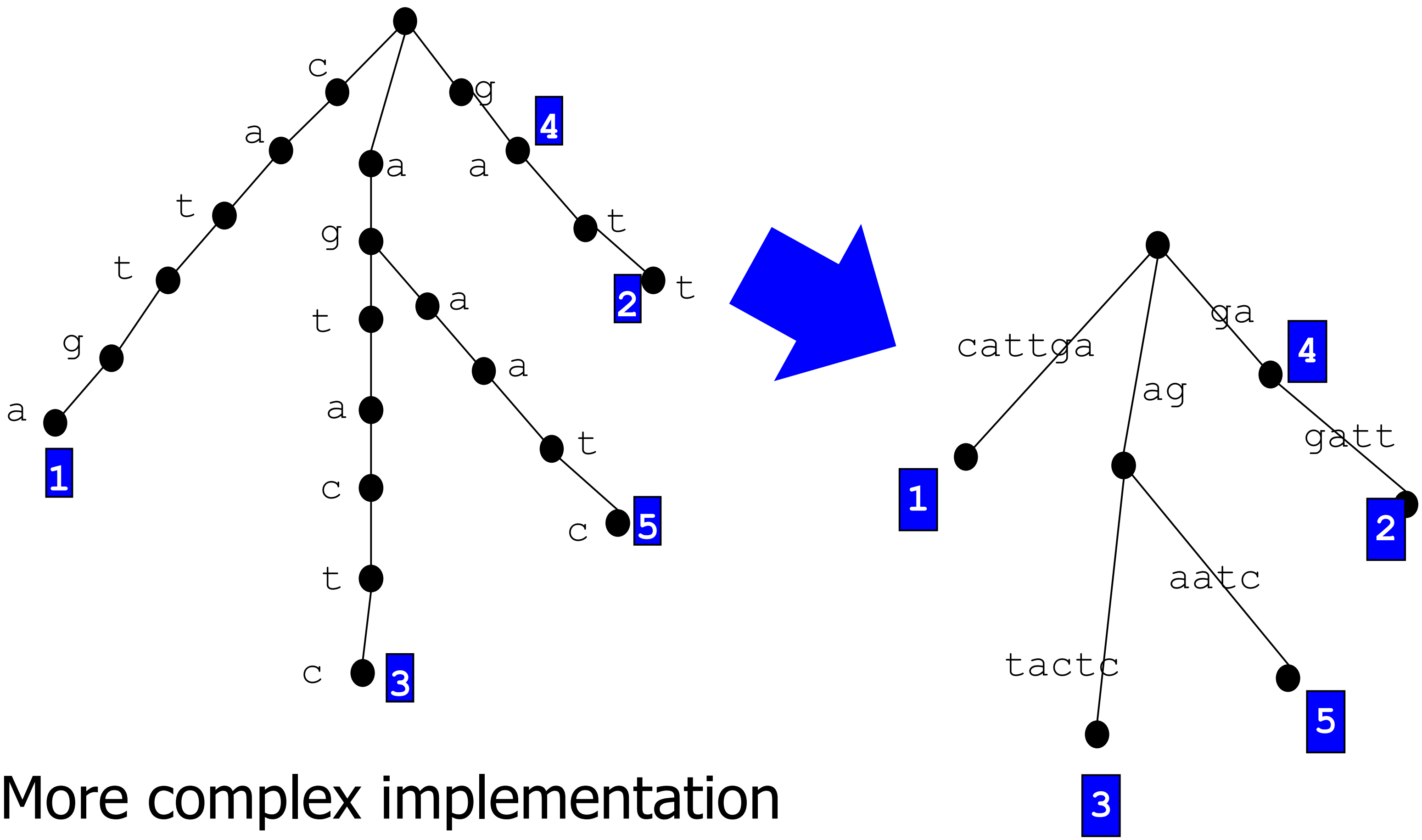
- Search t in S
- Recursively match t with a path starting from root
  - If no further match: $t \notin S$
  - If matched completely: $t \in S$

Search t="agtcc"



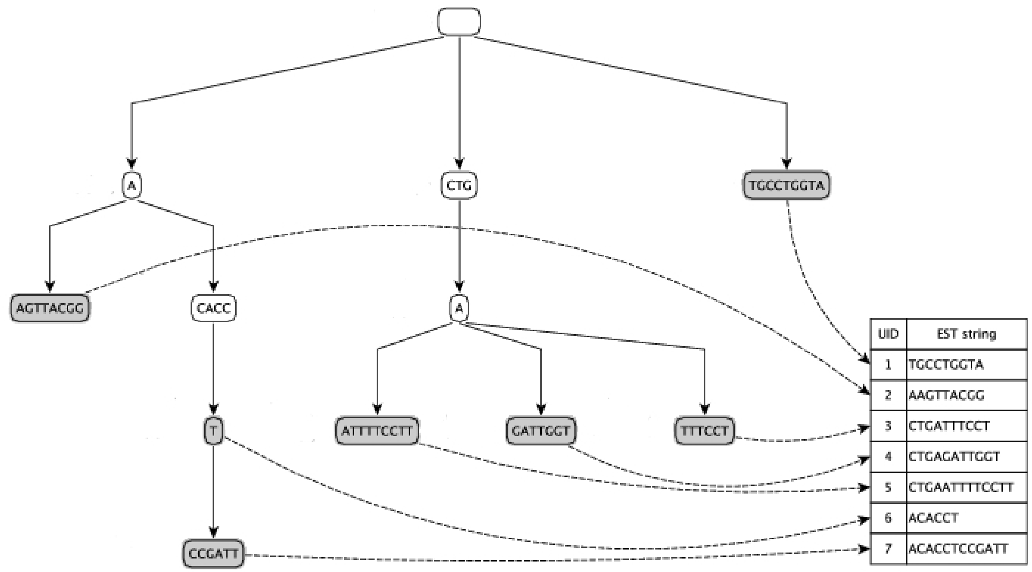
- Search complexity
  - Only depends on depth of S
  - Independent from |S|

Compressed Prefix Trees



- More complex implementation
- Different kinds of edges/nodes

Large Prefix Trees



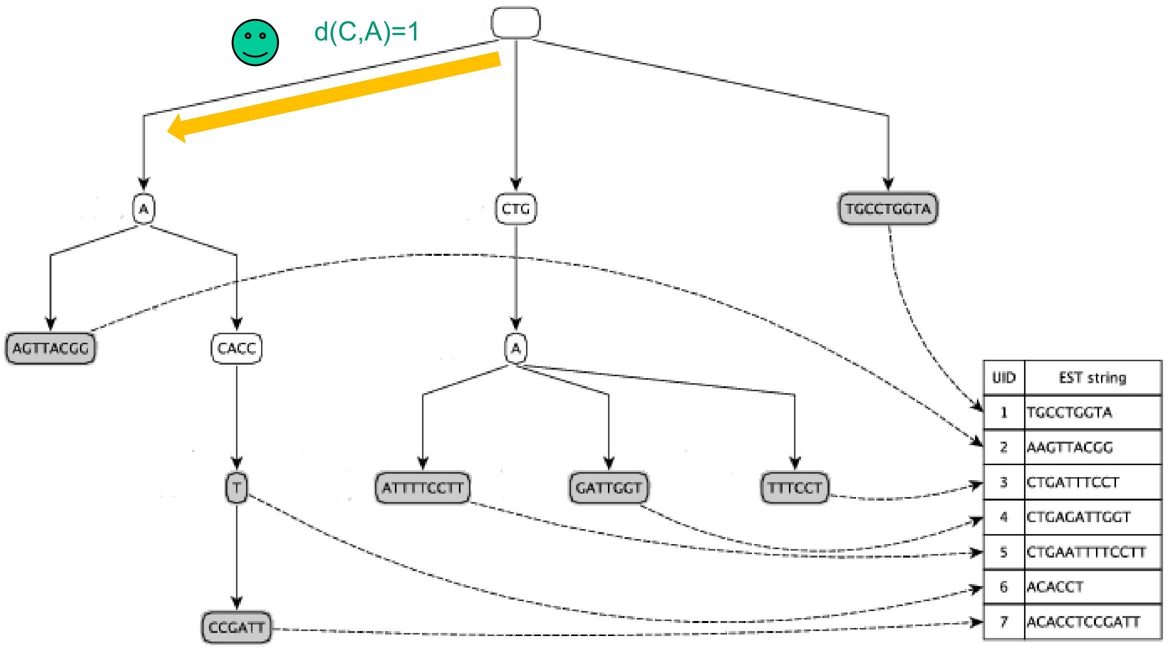
- Unique suffixes are stored (sorted) on disk
- Tree of common prefixes is kept in main memory
  - Most failing searches never access disc
  - At most one disc IO per search
  - [If tree fits in main memory]

Similarity Search on Prefix-Trees

- In similarity search, a mismatch doesn't mean that $t \notin S$
- Several mismatches might be allowed
  - Depending on error threshold
  - Depending on similarity function
- Idea
  - Depth-first search on the tree as usual
  - Keep a counter for the n# of errors occurring in the prefix so far
  - If counter exceeds threshold – stop search in this branch
  - Pruning: Try to stop earlier by clever "guessing"

Example: Search

Hamming distance search for t = CTGAAATTGGT, k=1



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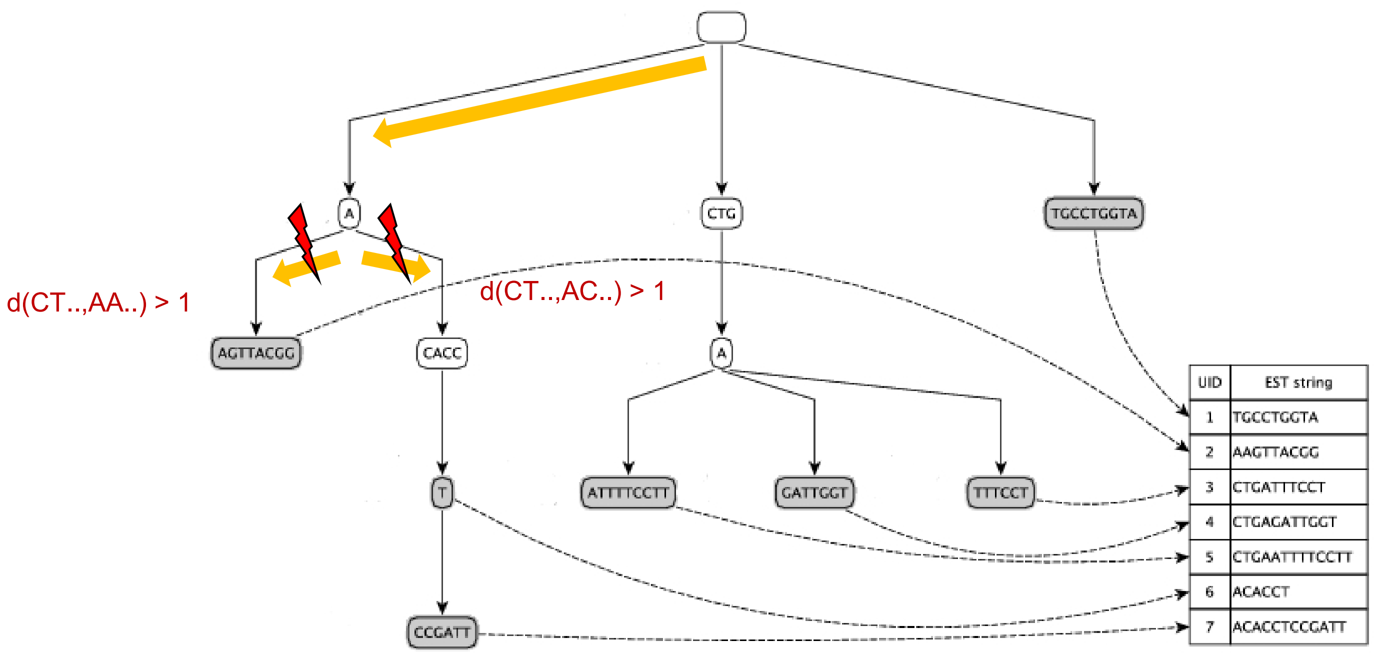

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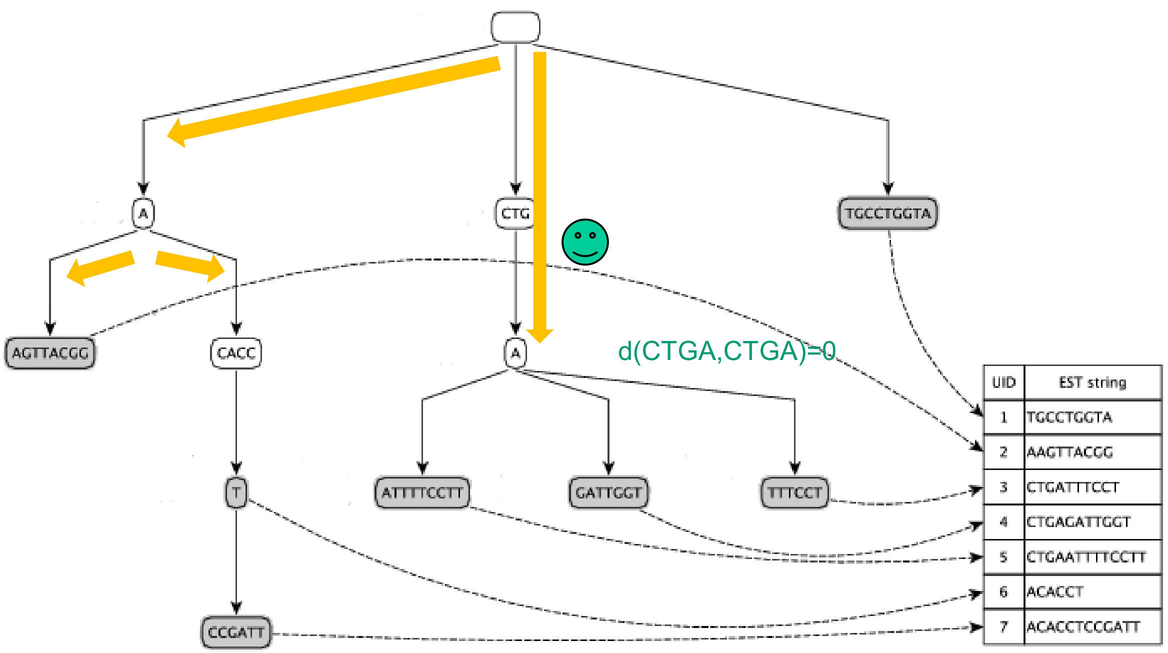

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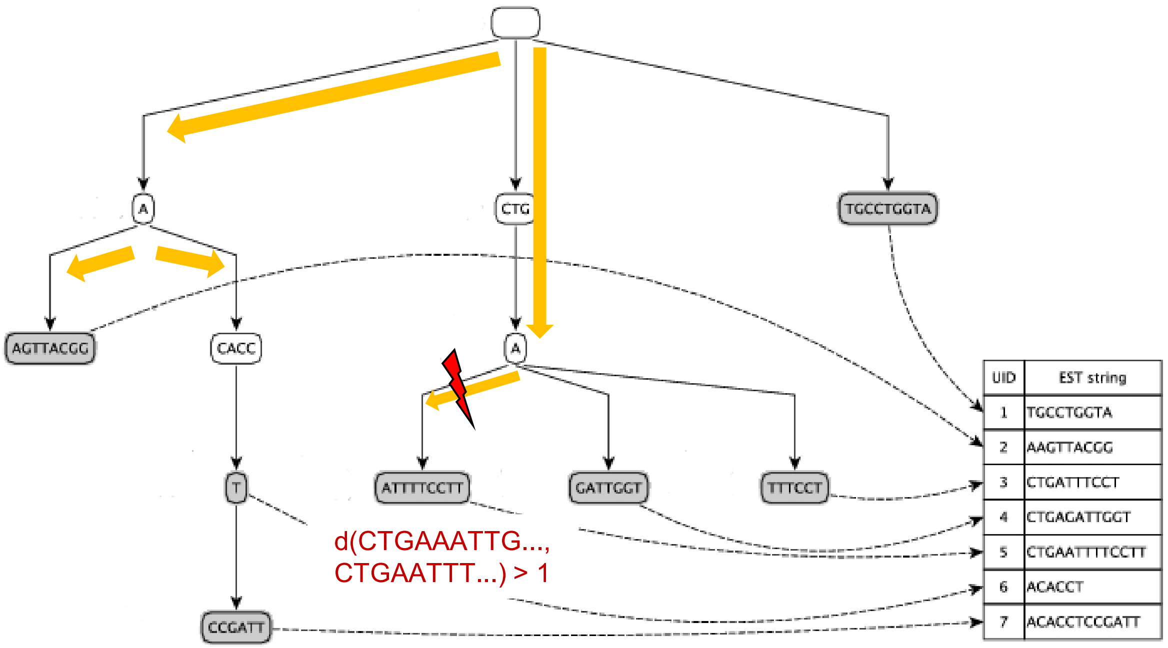

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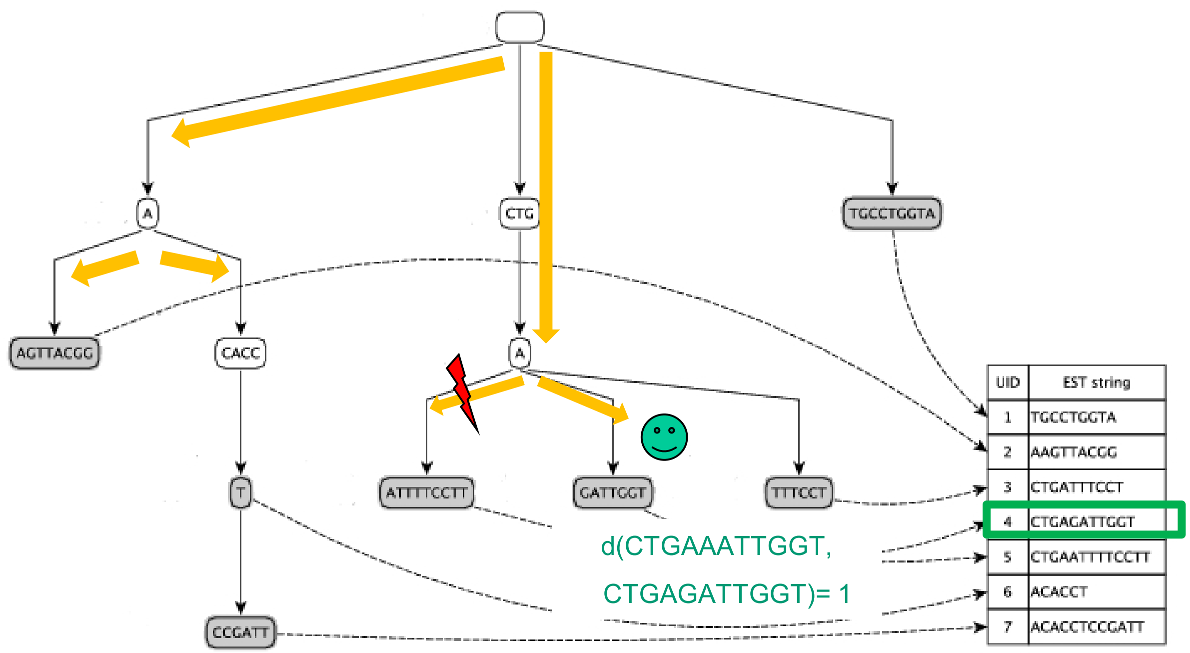

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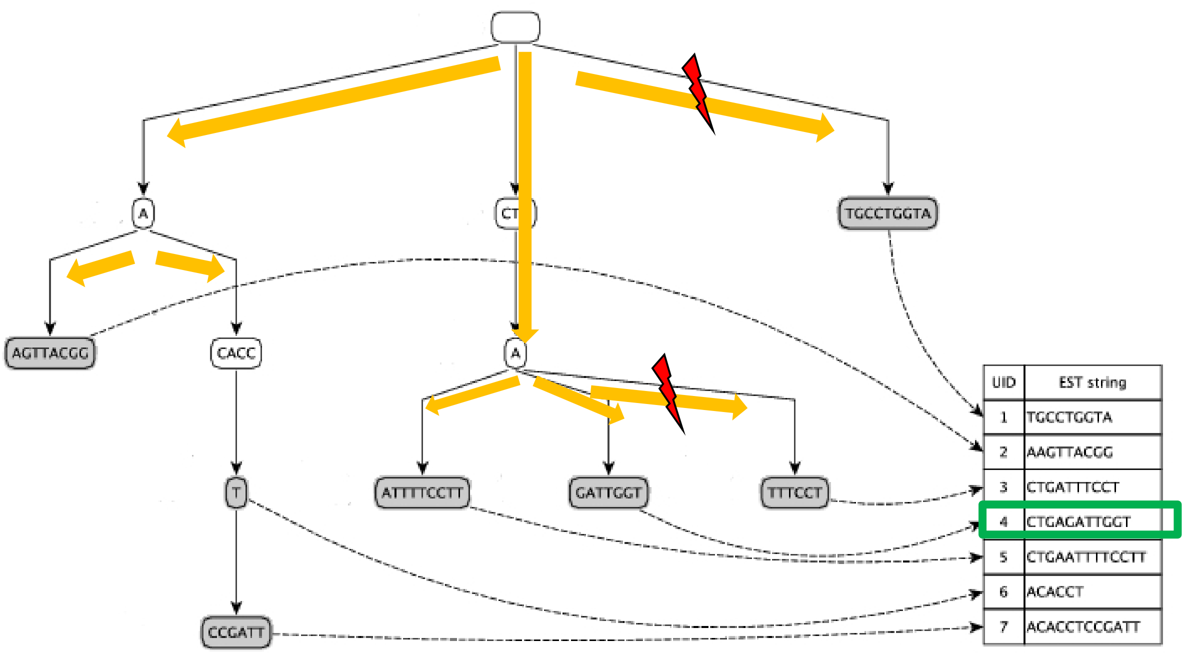
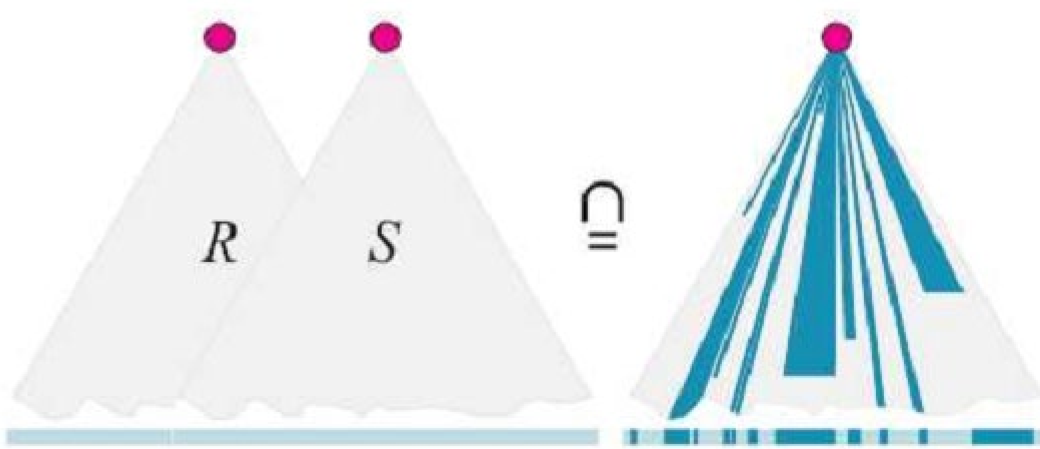


(Similarity) Joins on Prefix Trees

- We compare growing prefixes with growing prefixes
- Exact and similarity join
- Essentially: Compute intersection of two trees
  - Only labeled nodes are interesting
- Traverse both trees in parallel
  - Upon (sufficiently many) mismatches, entire subtrees are pruned



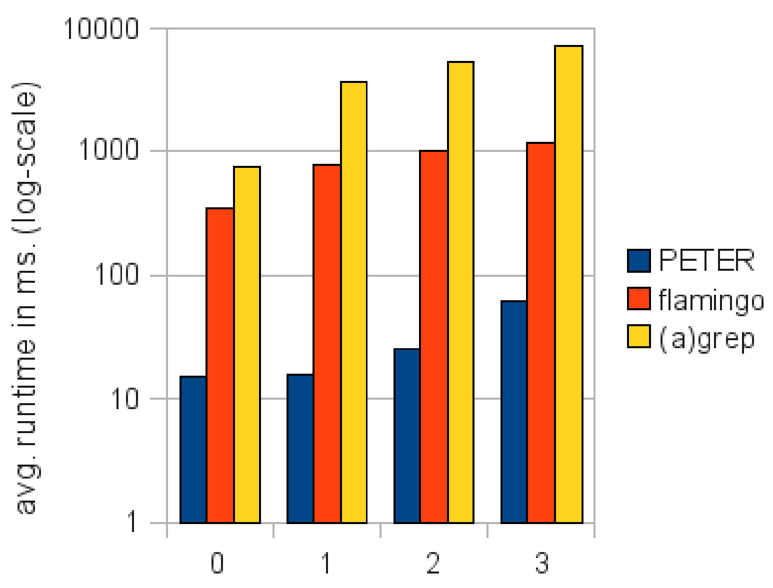
Evaluation

Set	# EST strings	avg. string length	min/max length	# tree nodes	# ext. suffixes
T_1	307,542	348	14/3,615	589,062	293,764
T_2	736,305	387	12/3,707	1,482,709	689,590
T_{2a}	368,152	382	12/2,774	711,632	352,872
T_{2b}	184,076	385	22/2,774	349,329	177,846
T_{2c}	92,038	383	25/2,774	171,964	89,198
T_{2d}	46,019	381	28/2,774	84,954	44,716
T_{2e}	23,009	373	31/ 878	42,375	22,366
T_3	10,000	536	16/3,707	16,310	8,774
TX	5,000,000	359	14/3,247	10,478,214	4,834,231

- Data: Several EST data sets from dbEST
  - Search: All strings of one data set in another data set
  - Join: One data set against another data set
  - Varying similarity thresholds
- (Linear) Index creation not included in measurements

Search: Comparing to Flamingo (2011)

- Flamingo: Library for approximate string matching
  - http://flamingo.ics.uci.edu/
  - Based on an inverted index on q-grams
  - Uses length and charsum filter

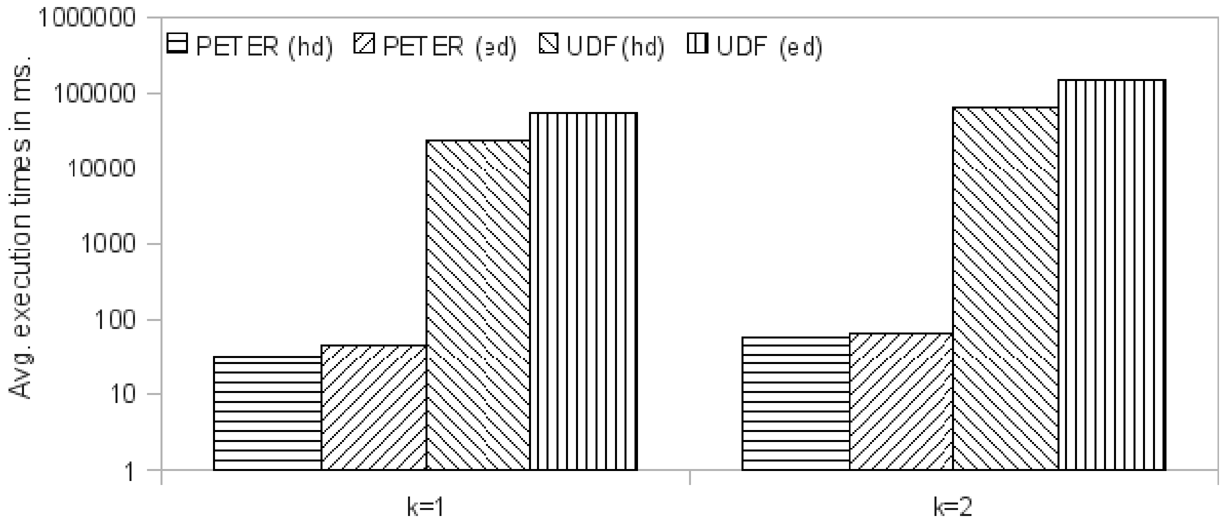


PETER inside a RDBMS

- We integrated PETER into a commercial RDBMS using its extensible indexing interface
  - Joins: table functions
  - Tree stored in separate file, suffixes stored in table
- Hope
  - As search complexity is independent of |S|, …
    - we might beat B+ trees for exact search on very large |S|
    - we might beat hash/merge for exact join of very large data sets
- First hope not fulfilled
  - API does not allow caching of tree – index reload for every search
  - Large penalty for context switch through API
    - Especially for JAVA!

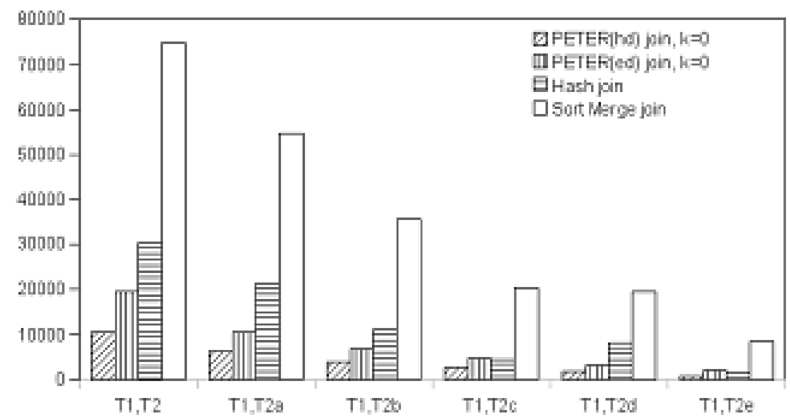
String Similarity Search in a RDBMS

- Peter (behind extensible indexing interface) versus UDF implementing hamming / edit distance calculations
- Difference: 2-3 orders of magnitude, independent of data set, threshold, or search pattern length



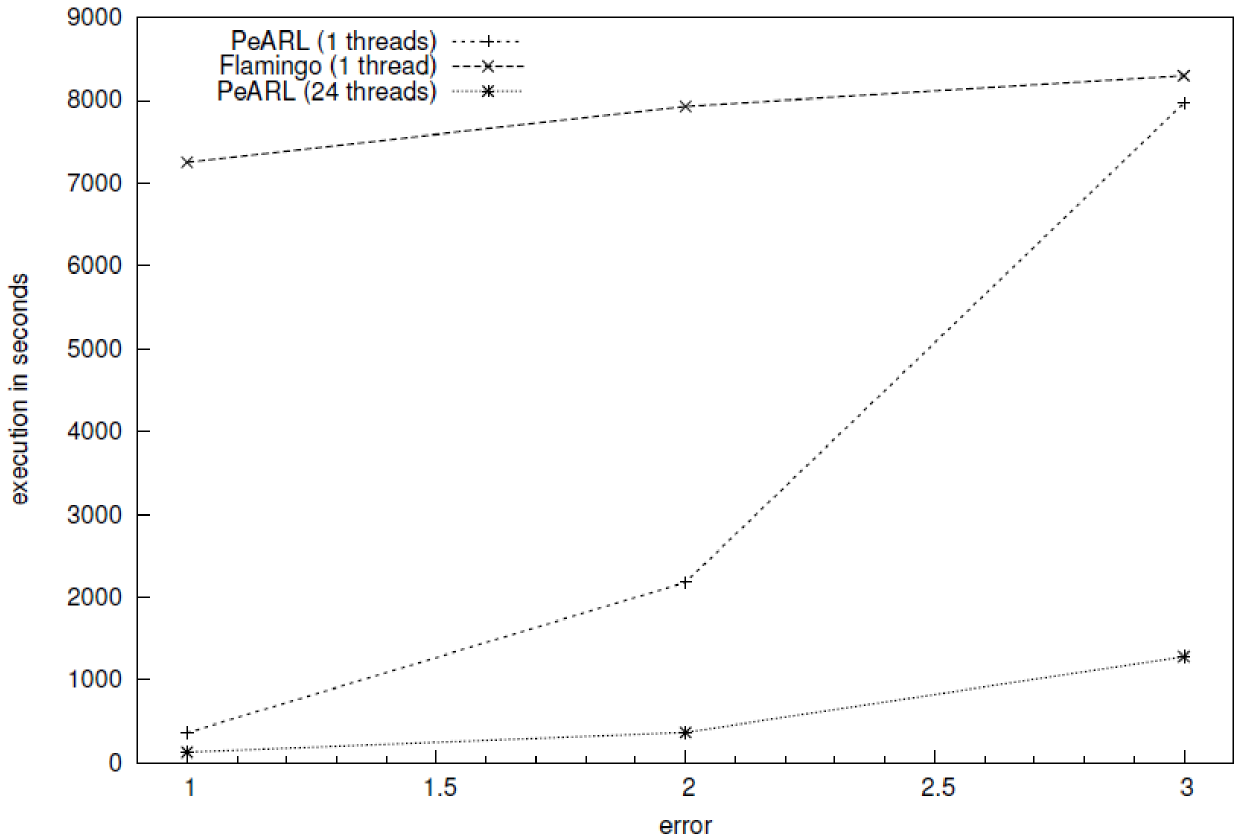
(Similarity) Join inside RDBMS

- PETER (behind extensible indexing interface) versus build-in join (exact join, hash and merge) or UDF
- Similarity join
  - Join T3 with T2e, k=2, inside RDBMS: Stopped after 24 h
  - Same join with PETER: 1 minute
- Exact join
  - For long strings, PETER is significantly faster than commercial join implementations



(b) Join

PEARL: Multi-Threaded PETER



Rheinländer, A. and Leser, U. (2011), “Scalable Sequence Similarity Search and Join in Main Memory on Multi-Cores”, HiBB, Bordeaux, France.

Room for Improvement

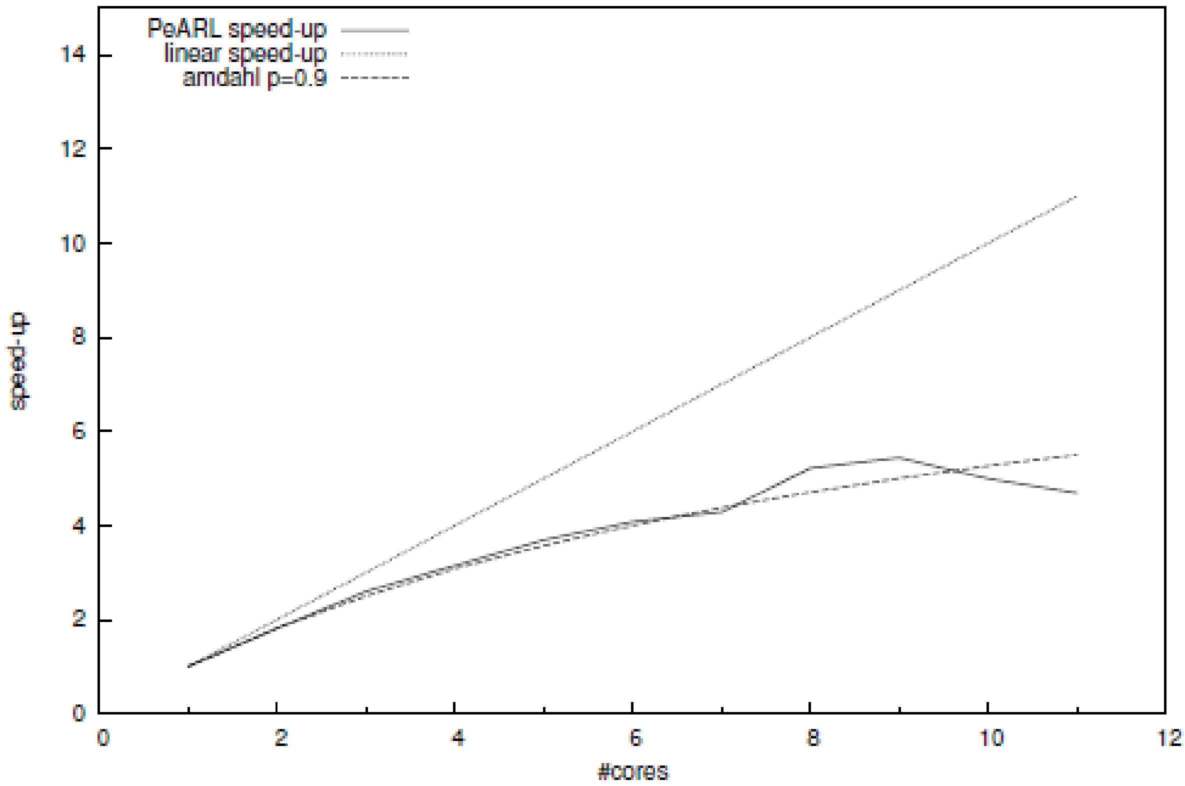


Fig. 7. PeARL speed-up for similarity search on k=2.

Why?

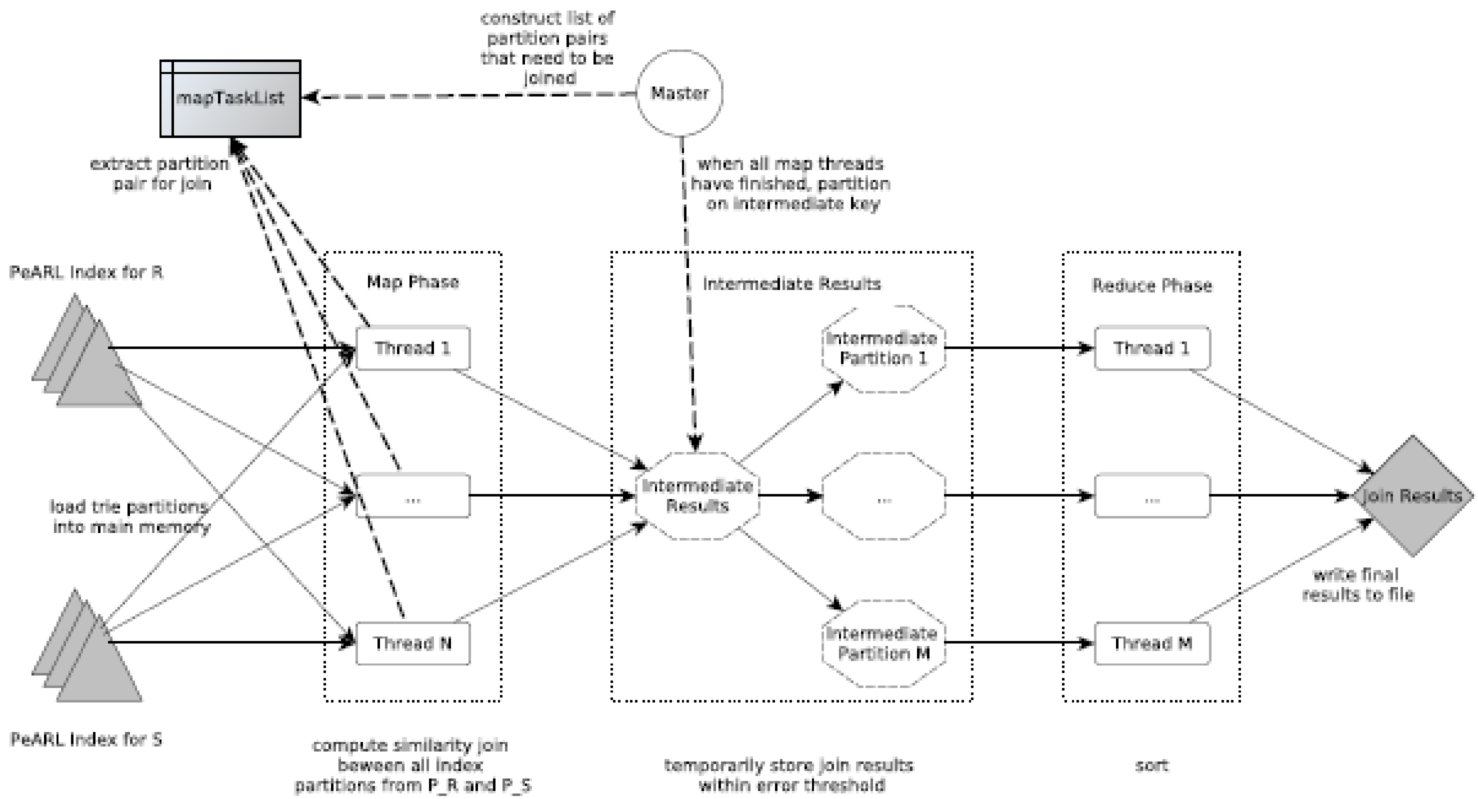


Fig. 2. MapReduce workflow of similarity joins in PeARL.