

# Maschinelle Sprachverarbeitung

Language Models

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### Content of this Lecture

- Language Models
- Markov Models
- Data sparsity
- Language Models for IR
- Most material from [MS99], Chapter 6

#### Problem

- Given a prefix of a sentence: Predict the next word
  - "At 5 o'clock, we usually drink ..."
    - "tea" quite likely
    - "beer" quite unlikely
    - "a beer" slightly more likely, but still
    - "biscuits" semantically wrong
    - "the windows need cleaning" syntactically wrong
- Similar to Shannon's Game: Given a series of characters, predict the next one (used in communication theory)
- Abstract formulation: Given a language L and the prefix S[1..n] of a sequence S, S∈L: Predict S[n+1]
- This is a ranking problem no single solution

### **Applications**

- Speech/character recognition
  - Given a transcribed prefix of a sentence which word do we expect next?
- Automatic translation
  - Given a translated prefix of a sentence what do we expect next?
- T9: "... information about common word combinations can also be learned ..."
- General: Use probabilities of next word as a-priori probability for interpreting the next signal
  - Helps to disambiguate between different options
  - Helps to make useful suggestions
  - Helps to point to likely errors (observation ≠ expectation)

### Language Models

- Usual approach: Learn a model of the language
- Classical model: (Deterministic) Grammars
  - Regular, context-free, ...
  - Grammars can be learned from examples
    - Not trivial, underdetermined, not covered here
  - Typical language grammars only determine syntactically correct continuations but disregard semantics
  - Usually, many continuations of a prefix are allowed
  - (Deterministic) Grammars cannot decide upon the most likely one

#### N-Grams over Words

- Popular and simple approach: N-gram models
  - "Indeed, it is difficult to beat a trigram model on the purely linear task of predicting the next word" [MS99]
- Definition
   A (word) n-gram is a sequence of n words.
- Usage
  - Given a corpus, perform sentence splitting and tokenization
  - Count frequencies of all n-grams in this corpus
    - Slide window of size n over sentences and keep counter for each ngram ever seen
  - Given a sentence prefix, predict most probable continuation(s)
     based on n-gram frequencies how?

# N-Grams for Language Modeling

- Assume a sentence prefix with n-1 words < w<sub>1</sub>,...,w<sub>n-1</sub> >
- Look-up counts of all n-grams starting with <w<sub>1</sub>,...,w<sub>n-1</sub>>
  - I.e., n-grams <  $w_1,..., w_{n-1}, w_x >$
- Choose w<sub>x</sub> whose n-gram is the most frequent one
- More formally
  - Compute, for every possibly w<sub>x</sub>,

$$p(w_x) = p(w_x \mid w_1, ..., w_{n-1}) = \frac{p(w_1, ..., w_x)}{p(w_1, ..., w_{n-1})}$$

Choose w<sub>x</sub> which maximizes p(w<sub>x</sub>)

#### Which n?

- In language modeling, one usually chooses n=3-4
- Seems small, but most language effects are local
  - But not all:
  - "The day I went to school the first time began with ..."
  - "... I went to school the first time when I was ..."
- Also, we cannot obtain robust relative counts for larger n not enough training data
  - "Data sparsity" problem (see later)
  - In high dimensional problems, training data is always sparse
  - Problem 1: We have no fitting n-grams because we haven't seen the n-1 prefix in our corpus – no prediction possible
  - Problem 2: Almost all n-grams get a count of 1 we cannot decide which continuation to predict

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### History and Applications

- Andrej Andrejewitsch Markov (1856-1922)
  - Russian Mathematician
  - Developed Markov Models (or Markov Chains) as a method for analyzing language
  - Markov, A. A. (1913). "Beispiel statistischer Untersuchungen des Textes "Eugen Onegin", das den Zusammenhang von Ereignissen in einer Kette veranschaulicht (Original in Russisch)." *Bulletin de l'Academie Imperiale des Sciences de St.-Petersbourg*: 153-162.
- Markov Models and Hidden Markov Models are popular
  - Language Modeling, Part-of-speech tagging
  - Speech recognition
  - Named entity recognition / information extraction
  - Biological sequence analysis

#### Markov Models

#### Definition

Assume an alphabet  $\Sigma$ . A Markov Model of order 1 is a sequential stochastic process with  $|\Sigma|$  states  $s_1, ..., s_n$  with

- Every state emits exactly one symbol  $w_i$  from  $\Sigma$
- No two states emit the same symbol
- For a sequence  $\langle s_1, s_2, ... \rangle$  of states, the following holds  $p(w_n = s_n / w_{n-1} = s_{n-1}, w_{n-2} = s_{n-2}, ..., w_1 = s_1) = p(w_n = s_n / w_{n-1} = s_{n-1})$
- For all s<sub>i</sub>, it holds that

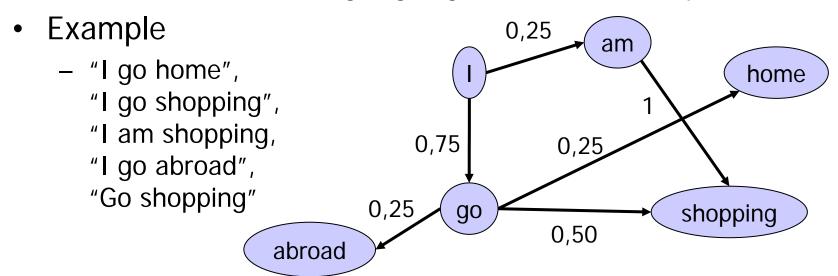
$$\sum_{s_{i+1}} p(w_{i+1} = s_{i+1} \mid w_i = s_i) = 1$$

#### Remarks

- $a_{i,j} = p(w_n = s_j | w_{n-1} = s_i)$  are called transition probabilities
- In language modeling,  $\Sigma = \{\text{set of all words of a language}\}$
- Computing good start probabilities is an issue we essentially ignore

#### Visualization

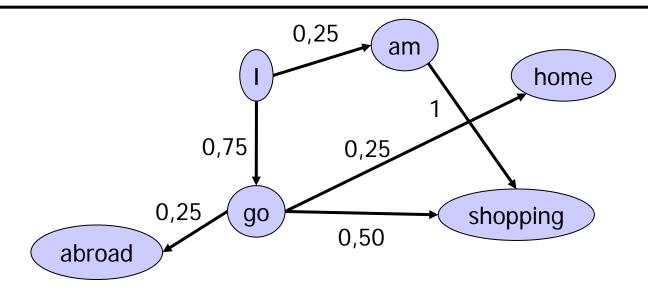
- Since every state emits exactly one word, we can merge states and words
- State transition graph
  - Nodes are states (labeled with their emission)
  - Arcs are transitions labeled with a non-zero probability
  - Probabilities of all outgoing edges of a state sum up to 1



# Probability of a Sequence of States (=a Sentence)

- Assume a Markov Model M and a sequence S of states with |S|=n
- With which probability was S generated by M, i.e., what is the value of p(S|M)?

$$p(S \mid M) = p(w_1 = S[1]) * \prod_{i=2..n} p(w_i = S[i] \mid w_{i-1} = S[i-1])$$
$$= a_{0,S[1]} * \prod_{i=2..n} a_{S[i-1],S[i]} = a_{0,1} * \prod_{i=2..n} a_{i-1,i}$$



- p("I go home") =  $p(w_1 = {}_{n}I"|w_0)^* p(w_2 = {}_{n}go"|w_1 = {}_{n}I")^* p(w_3 = {}_{n}home"|w_2 = {}_{n}go")$ = 1 \* 0.75\* 0.25 = 0.1875
- Problem: Pairs not in the training data get probability 0
  - Example: "I am abroad"
  - With this small "corpus", almost all transitions get p=0

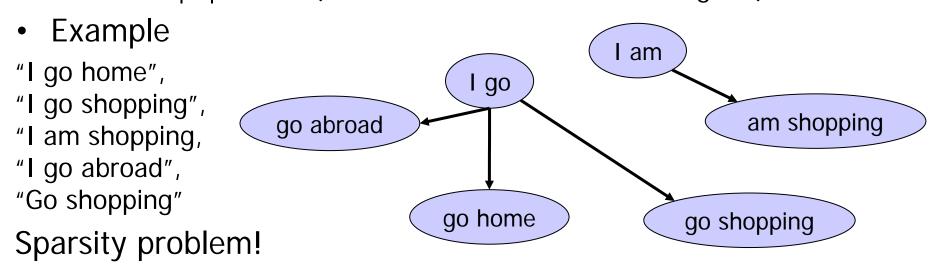
#### **Stochastic Processes**

- Consider language generation as a sequential stochastic process
- At each stage, the process generates a new word
  - Like a DFA, but transitions have probabilities
- Question: How big is the memory? How many previous words does the process use to determine the next step?
  - 0: Markov model order 0: No memory at all
  - 1: Markov model order 1: Next word only depends on prev. word
  - 2: Markov model order 2: Next word only depends on 2 prev. words
  - **–** ...

### Higher Order Markov Models

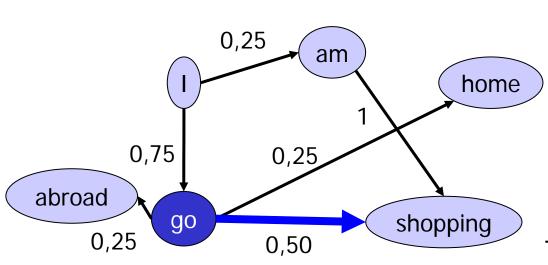
• Markov Models of order k: The prob. of state s after n steps depends on the k predecessor states  $s_{n-1},...,s_{n-k}$   $p(w_n=s_n/w_{n-1}=s_{n-1}, w_{n-2}=s_{n-2},..., w_1=s_1) = p(w_n=s_n/w_{n-1}=s_{n-1},..., w_{n-k}=s_{n-k})$ 

- We can transform any order k model M (k>1) into a Markov Model of order 1 (M')
  - M' has |M|k states (all combinations of states of length k)



### Predicting the Next State

- For language modeling, we do not need the probability of an entire sequence, but we only reason about the next state given some previous states
- Consider an order-1 Markov Model



$$p(w_n) = p(w_n | w_1, ..., w_{n-1})$$

$$= p(w_n | w_{n-1})$$

$$= \frac{p(w_{n-1}, w_n)}{p(w_{n-1})}$$

$$\sim p(w_{n-1}, w_n)$$

This is the most frequent bi-gram with prefix w<sub>n-1</sub>

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#### Problem

- We learn our transition probabilities from a limited sample
- Thus, we only estimate the true transition probabilities
- Introduces a systematic error which we can try to alleviate
  - Sample selection is important
  - Problem is researched a lot in statistics.
- Extreme cases: Transitions we do not see in the corpus
  - Get a probability of 0
  - Will never be predicted
  - This does not mean that they are non-existing in the language
- Our model (yet) cannot adequately cope with data sparsity

### Importance of Data Sparsity

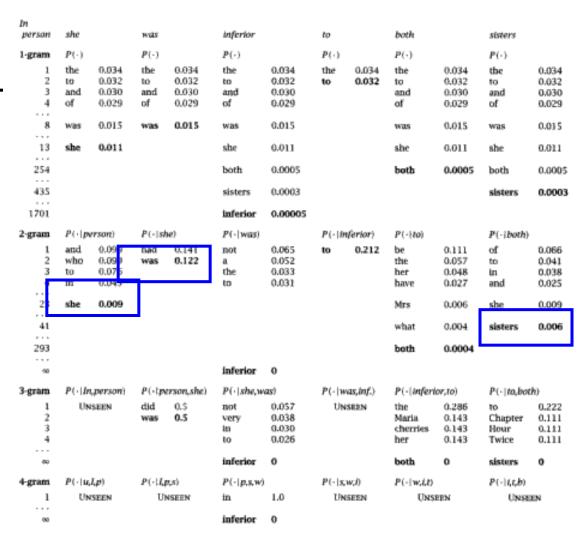
- How many n-grams do exist in principle?
  - Assume a language of 20.000 words
  - n=1: 20.000, n=2: 4E8, n=3: 8E12, n=4: 1.6E17, ...
  - Very bad estimates: Natural languages have many more words, but most combinations are not allowed
- In "normal" corpora over natural languages, almost all ngrams with n>4 are very sparse
  - Exponential growth cannot be balanced by "use larger corpora"
  - Rare (and therefore very specific) n-grams are missed
- Trade-off: N-gram models
  - Large n: More expressive model, but bad estimations
  - Small n: Less expressive model, but better estimates

Unigrams (order 0):
 Always the most frequent word in the corpus, does not differentiate

In person	she		was		inferior		to		both		sisters		
1-gram	$P(\cdot)$		P(-)		P(+)		$P(\cdot)$		P(-)		P(-)		
1	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034	the	0.034	
3 4	and of	0.030 0.029	and of	0.030	and of	0.030 0.029			and of	0.030 0.029	and of	0.030	
8	was	0.015	was	0.015	was	0.015			was	0.015	was	0.015	
13	she	0.011			she	0.011			she	0.011	she	0.011	
254					both	0.0005			both	0.0005	both	0.0005	
435					sisters	0.0003					sisters	0.0003	
1701					inferior	0.00005							
2-gram	$P(\cdot person)$		$P(\cdot she)$		$P(\cdot was)$	P(- was)		$P(\cdot inferior)$		$P(\cdot to)$		$P(\cdot both)$	
1 2 3 4	and who to in	0.099 0.099 0.076 0.045	had was	0.141 0.122	not a the to	0.065 0.052 0.033 0.031	to	0.212	be the her have	0.111 0.057 0.048 0.027	of to in and	0.066 0.041 0.038 0.025	
23	she	0.009							Mrs	0.006	she	0.009	
41									what	0.004	sisters	0.006	
293									both	0.0004			
100					inferior	0							
3-gram	$P(\cdot In,person)$		P(-tperson,she)		$P(\cdot   she, was)$		$P(\cdot was,inf.)$		$P(\cdot inferior,to)$		$P(\cdot   to, both)$		
1 2 3 4	Unseen		did was	0.5 <b>0.5</b>	not very in to	0.057 0.038 0.030 0.026	Uns	HEN	the Maria cherries her	0.286 0.143 0.143 0.143	to Chapter Hour Twice	0.222 0.111 0.111 0.111	
80					inferior	0			both	0	sisters	0	
4-gram	$P(\cdot   u_i I_i p)$		$P(\cdot I_ip_is)$		$P(\cdot p,s,w)$		$P(\cdot   y, w, t)$		P(- w,i,t)		P(- 1,t,b)		
1	Unseen		Unseen		in.	1.0	Uns	SEEN	Unseen		UNSEEN		
90					inferior	0							

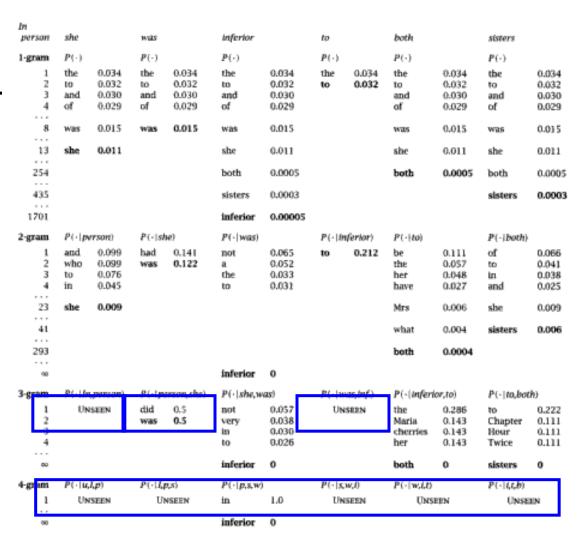
**Table 6.3** Probabilities of each successive word for a clause from *Persuasion*. The probability distribution for the following word is calculated by Maximum Likelihood Estimate *n*-gram models for various values of *n*. The predicted likelihood rank of different words is shown in the first column. The actual next word is shown at the top of the table in italics, and in the table in bold.

 Bi-grams (order 1): Correct word often ranks high, but not always



**Table 6.3** Probabilities of each successive word for a clause from *Persuasion*. The probability distribution for the following word is calculated by Maximum Likelihood Estimate *n*-gram models for various values of *n*. The predicted likelihood rank of different words is shown in the first column. The actual next word is shown at the top of the table in italics, and in the table in bold.

- Tri-grams (order 2):
   Has a 50% hit, but already suffers from sparsity (unseen)
- Four-grams: Unusable
- Corpus: Fraction of Jane Austen's oeuvre, ~600.000 tokens, data from [MS99]



**Table 6.3** Probabilities of each successive word for a clause from *Persuasion*. The probability distribution for the following word is calculated by Maximum Likelihood Estimate *n*-gram models for various values of *n*. The predicted likelihood rank of different words is shown in the first column. The actual next word is shown at the top of the table in italics, and in the table in bold.

#### Some Practical Solutions

- Reduce the number of words using stemming
  - Might help to go from n=3..4 to n=4...5
  - Important grammatical clues are lost
- More abstract: Use some form of "binning" of tokens into classes and compute n-grams over token classes, not token
  - All numbers -> one class
  - All verbs -> one class (POS tags)
  - All verbs related to "movement" -> one class

#### Statistical Estimators

- We were a bit sloppy so far
- We want

$$p(w_n) = p(w_n \mid w_1, ..., w_{n-1}) = \frac{p(w_1, ..., w_n)}{p(w_1, ..., w_{n-1})}$$

- But we only have  $count(w_1,...,w_n)$
- So far, we always implicitly assumed

$$p(w_1,...,w_n) = \frac{count(w_1,...,w_n)}{N}$$

N: all observed n-grams

# MLE for N-gram Models

- This is called a Maximum Likelihood Estimator (MLE)
- MLE gives maximum likelihood to the training data
  - Gives zero probability to all events not in the training data
  - The probability mass is spent entirely on the training data
  - Gives optimal results when applied to the training data
  - Overfitting
- Need to smooth the estimates to account for the limitations of the sample

### Smoothing I: Laplace's Law

- Give some probability mass to unseen events
- Oldest (and simplest) suggestion: "Adding one"

$$p_{LAP}(w_1,...,w_n) = \frac{count(w_1,...,w_n)+1}{N+B}$$

- Where B is the number of possible n-grams, i.e., K<sup>n</sup>
- All n-grams get a probability≠0
- But moves too much mass to the unknown
  - Estimates for seen n-grams are scaled down dramatically
  - Estimates for unseen n-grams are small, but there are so many
    - And many of them are truly impossible
  - In a corpus of 40 M words with K~400T, 99.7% of the total probability mass is spend in unseen events

### Smoothing II: Lidstone's Law

- Laplace not suitable if there are many events, but few seen
- Lidstone's law gives less probability mass to unseen events

$$p_{LIP}(w_1,...,w_n) = \frac{count(w_1,...,w_n) + \lambda}{N + \lambda * B}$$

- Small  $\lambda$ : More mass is given to seen events
- Typical estimate is  $\lambda = 0.5$
- Appropriate values can be learned (next slide)
- Still: Estimate of seen events is linear in the MLE estimate
  - Not a good approximation of empirical distributions
- Other: Good-Turing Estimator, n-gram interpolations, ...

### Learning Appropriate Values for $\lambda$

- We "simulate" seen and unseen events
- Divide corpus in two disjoint parts C<sub>1</sub> and C<sub>2</sub>
- Count frequencies of n-grams in C<sub>1</sub>
- Let c be the number of n-grams from C<sub>1</sub> not present in C<sub>2</sub>
- Set  $\lambda = c/B$ 
  - The probability of an n-gram (in C<sub>2</sub>) to be considered as not existing although in reality it does exist

### Option III: Back-Off Models

- If we cannot find a n-gram with count≠0, use a (n-1)-gram
  - Or an n-2 gram, ...
- Thus, in case there is no p(w<sub>1</sub>,...,w<sub>n</sub>)≠0, we "back off" to a simpler model

$$p(w_n \mid w_1, ..., w_{n-1}) = \frac{p(w_1, ..., w_n)}{p(w_1, ..., w_{n-1})} \approx \frac{p(w_2, ..., w_n)}{p(w_2, ..., w_{n-1})} \approx \frac{p(w_3, ..., w_n)}{p(w_3, ..., w_{n-1})} \approx ...$$

- Stop at the first (n-k)-gram with non-zero count
- Alternative: Always look at different n's
  - With different weights

$$p(w_n) = \lambda_1 \frac{p(w_{n-2}, w_{n-1}, w_n)}{p(w_{n-2}, w_{n-1})} + \lambda_2 \frac{p(w_{n-1}, w_n)}{p(w_{n-1})} + \lambda_3 p(w_n)$$

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#### New IR Model

- Recent trend in IR: Relevance based on language models
- Idea: See a document as a "language"
  - Learn a model of this language
  - See with which probability this model has generated the query
  - Rank documents based on these probabilities
- Sounds weird, but leads to a simple and well justified approach
- Very successful in recent evaluations
- Smoothing is crucial docs are too small

### **Approach**

- As docs are small, only unigram models are sensible
- Model of a doc: Relative frequencies of all its words
- Compute

$$p(d | q) = \frac{p(q | d) * p(d)}{p(q)} \sim p(q | d) * p(d) \sim p(q | d)$$

- p(q) is equal for all d irrelevant for ranking
- p(d) can be used to incorporate a-prior knowledge (e.g. prestige),
   but often is set to uniform irrelevant for ranking
- We replace d with its model and obtain

$$p(q \mid d) = p(q \mid M_d) = p(k_1, k_2, ..., k_n \mid M_d) = \prod_{k \in q} p(k \mid M_d) = \prod_{k \in q} \frac{tf_{k,d}}{\mid d \mid}$$

#### Discussion

- Very simple
- Principled approach to justify usage of tf values
- More powerful for longer queries
- Problems
  - Words in q not in d: Smoothing
  - Where is idf gone?

### Smoothing a Language Model for IR

- For instance, if  $k \notin d$ , set  $p(k|M_d) = df_k/|D| = p(k|M_D)$ 
  - Token that are in d are counted with tf values (and not discounted with idf); tokens not in d are counted with df values
- More tunable parameters: Linear interpolation

$$p'(k | M_d) = \lambda * p(k | M_d) + (1 - \lambda) * p(k | M_D)$$

- Combine relevance of k in document and relevance of k in corpus
- Large  $\lambda$ : More weight to the document, less weight to background
- $-\lambda$  may vary, for instance with query size
- We are back at something similar to TF\*IDF, but with a probabilistic interpretation, not a geometric one

#### Self Assessment

- What is language modelling about?
- Define a Markov model
- How can you turn a Morkov model of order 4 into one of order 1?
- What is the data sparsity problem (in language modeling)?
- What is the disadvantage of Laplace smoothing?
- Explain how we can use language models for information retrieval